

Chain Rule

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## Chain Rule - Differentiation

Given functions  $f(x)$  and  $g(x)$  the derivative of the function,

$$F(x) = f(g(x)) = f \circ g(x)$$

is given by,

$$F'(x) = f'(g(x))g'(x) \quad (1)$$

Equation (1) is called the *chain rule* of differentiation.

### Example

Find the derivative of the function,

$$y = (x^3 + 1)^2 \quad (2)$$

**Solution:** We first need to determine what the two function  $f$  and  $g$  are in order to apply the chain rule (1). Taking a look at equation (2) if we take  $f(x) = x^2$  and  $g(x) = x^3 + 1$ , the composition  $f \circ g(x)$  gives us the function  $f \circ g(x) = (x^3 + 1)^2$ . So, now we can apply the chain rule and we know what functions  $f$  and  $g$  we are considering in equation (1).

$$\begin{aligned} y' &= 2(x^3 + 1)^1(3x^2) \\ &= 2(3x^2)(x^3 + 1) \\ &= 6x^2(x^3 + 1) \end{aligned}$$

Therefore,  $y' = 6x^2(x^3 + 1)$ .

## Exercises

Use the chain rule to differentiate the following functions.

a)  $y = (-5 - x^2)^3$

e)  $y = -(3x^2 + 2x^3)^{-1}$

b)  $y = (3 - 4x^3)^4$

f)  $y = \frac{1}{2x+6x^2}$

c)  $y = (1 + 2x)^{-2}$

g)  $y = \frac{1}{(x^2-x-1)^4}$

d)  $y = (2 + x + 3x^2)^{-4}$