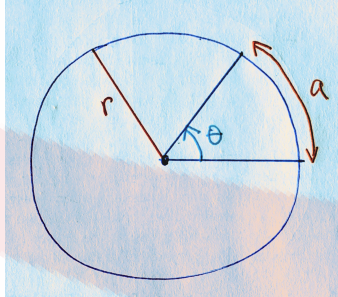Arc Length

Arc Length

What is **arc length**? The arc length is the length of the arc travelled around a circle with a given radius r for a given angle θ in radians. Let's consider the following diagram of a circle with raidus r.

A slice of pie or *sector* of the circle is cut out. The angle at the centre of the circle of the pie or setor is θ and the length of the outer edge or arc length of the sector is givem by a. The relationship between θ, a and r is given by,

$$\theta = \frac{a}{r} \tag{1}$$

Let's consider an example.

Example

Find the arc length of a sector of a circle with radius $r = 9cm$ with angle $45°$.

Solution: The angle $45°$ can be written as $\pi/4$ radians. Using equation (1) we can solve for a,

$$\begin{aligned}
\theta &= \frac{a}{r} \\
a &= r\theta
\end{aligned}$$

Now we can plug in our values for θ and r and evaluate for a.

$$\begin{aligned}
a &= (9)(\pi/4) \\
&= \frac{9\pi}{4}
\end{aligned}$$

Therefore, the arclength of the circle of radius $9cm$ with angle $45°$ is $\frac{9\pi}{4}cm$.

Exercises

Find the arc length travelled throughout the angle θ below along a circle with the radius r given below.

1. $\theta = \frac{7\pi}{6}$, $r = 3$

2. $\theta = \frac{3\pi}{4}$, $r = 4$

3. $\theta = \frac{\pi}{4}$, $r = 2$

4. $\theta = \frac{\pi}{2}$, $r = 5$

5. $\theta = \frac{\pi}{5}$, $r = 6$

6. $\theta = \frac{4\pi}{3}$, $r = 2$

7. $\theta = 2\pi$, $r = 5$

8. $\theta = \frac{6\pi}{11}$, $r = 3$

9. $\theta = \pi$, $r = 8$

10. $\theta = \frac{5\pi}{7}$, $r = 9$