Vector equation of a line.

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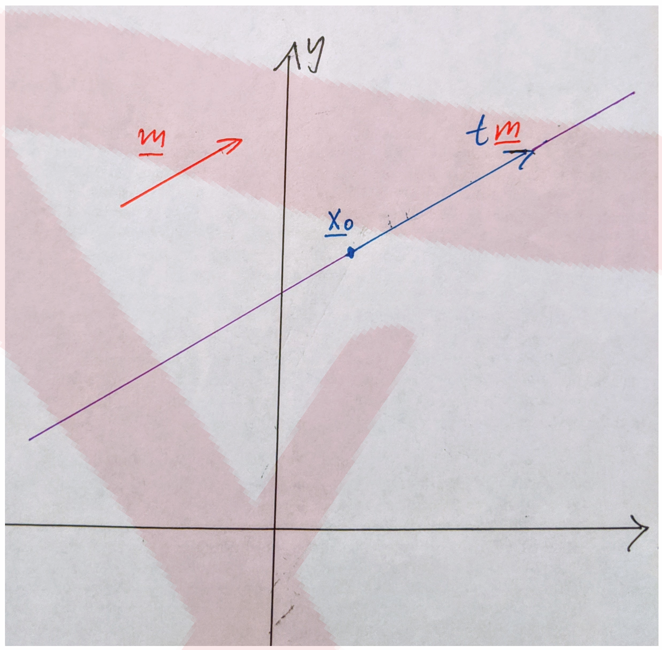
Vector equation of a line in $\mathbb{R}^2$

Suppose we are given that a line passes through the point $\underline{x}_0 = (x_{01}, x_{02})$ and points in the direction $\underline{m} = (m_1, m_2)$. Both $\underline{x}_0$ and $\underline{m}$ are vectors. The vector equation of the line is,

$$\underline{x} = \underline{x}_0 + \underline{m}t$$

where, $\underline{m}t \in \mathbb{R}^2$ and t is any real number.



Similarly, the vector equation of a line in $\mathbb{R}^{\nvDash}$ is given by,

$$\underline{x} = \underline{x}_0 + \underline{m}t$$

where, $\underline{m}t \in \mathbb{R}^3$ and t is any real number.

Example in $\mathbb{R}^2$

Given the points P=(3,2), Q=(-1,2) and R = (-1, -2), find the vector equation of the line through the piont P in the direction of QR.

Solution We need to use the vector equation of a line,

$$(x, y) = (x_0, y_0) + (m_1, m_2)t$$

where t is any real number or $t \in \mathbb{R}$ and P=(3, 2) $= (x_0, y_0)$ and $\underline{m} = (m_1, m_2)$ is the direction of QR. We need to find $\underline{m}$.

$$\begin{aligned} \underline{m} &= R - Q = (-1,-2) - (-1,2) \\ &= (-1-(-1), -2-2) \\ &= (-1+1, -4) \\ \therefore \underline{m} &= (0,4) \end{aligned}$$

Therefore, $(x, y) = (3, 2) + (0, 4)t$.

Example in $\mathbb{R}^3$

Given the points P = (1, 0, -1), Q = (-2, 1, 1) and R = (0, 1, 3). Find the vector equation of the line through P in the direction of QR.

Solution The vector equation of a line in $\mathbb{R}^3$ is given by,

$$(x, y, z) = (x_0, y_0, z_0) + (m_1, m_2, m_3)t$$

where P=(1, 0, -1) $= (x_0, y_0, z_0)$ and

$$\begin{aligned} \underline{m} &= (m_1, m_2, m_3) = QR = R - Q \\ &= (0,1,2) - (-2,1,1) = (2,0,2) \end{aligned}$$

Therefore, $(x, y, z) = (1, 0, -1) + (2, 0, 2)t$ is the vector equation of the line, where t is any real number or $t \in \mathbb{R}$.

Exercises

Given that a line passes through P and in the direction QR find the vector equation of the lines below.

a) $P(3,2), Q(0,2), R(-3,1)$

b) $P(4,1), Q(-1,0), R(0,4)$

c) $P(-1,2,1), Q(2,0,-1), R(-1,3,0)$

d) $P(3,-2,0), Q(2,2,2), R(-1,2,-1)$

e) $P(-5,3), Q(0,1), R(5,4)$

f) $P(-2,0), Q(1,3), R(4,-1)$

g) $P(4,0,-3), Q(-1,2,0), R(1,1,1)$

h) $P(-1,-2,1), Q(4,3,-1), R(3,1,2)$

i) $P(-3,-4), Q(2,0), R(5,4)$

j) $P(0,-4,-1), Q(3,0,0), R(-2,3,1)$

k) $P(-2,5,-3), Q(2,2,-2), R(1,3,4)$