Solving Inequailties

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Solving Inequalities

Solving equalities involves expressions with an "=" sign in it. Solving *inequalities*, involve expressions with an inequality, $<, >, \leq$ or $\geq$ in them instead of the "=". Let's consider an example.

Example

Solve the following inequality,

$$0 \geq x^2 + 7x + 10$$

Solution

1. First we consider the "equality" or equation by replacing the inequality sign, $\geq$ in this case, with an "=" sign. Doing this we get,

$$0 = x^2 + 7x + 10$$

2. Now we follow the same steps as above for solving equations.

$$\begin{aligned} 0 &= x^2 + 7x + 10 \\ 0 &= (x+5)(x+2) \\ x+5 &= 0 \text{ or } x = 2 = 0 \\ x &= -5 \text{ or } x = -2 \end{aligned}$$

3. Let's re-insert the inequality sign $\geq$ and see what we get.

$$\begin{aligned} 0 &\geq x^2 + 7x + 10 \\ 0 &\geq (x+5)(x+2) \end{aligned}$$

When is the product of two numbers less than or equal to 0? Good question. If we think about this, the product of two numbers is less than 0 when exactly one nubmer is less than 0 and the product of two numbers is equal to 0 when at least one of the nubmers is equal to 0. In our case this means,

$$\begin{aligned} x+5 &\leq 0 \text{ or } x+2 \leq 0 \\ x &\leq -5 \text{ or } x \leq -2 \end{aligned}$$

4. The solution of the inequaltiy is given by,

$$\begin{aligned} 0 &\geq x^2 + 7X + 10 \\ 0 &\geq (x+5)(x+2) \end{aligned}$$

where $x \leq -5$ or $x \leq -2$, but not both.

Let's try another example.

Example

Solve the following inequality,

$$0 \leq x^2 + 11x - 12$$

Solution First, let's factor the right hand side.

$$\begin{aligned} 0 &\leq x^2 + 11x - 12 \\ 0 &\leq (x+12)(x-1) \end{aligned}$$

When is a product of two numbers greater than or equal to 0? When both values are positive, negative or at least one is 0. With this in mind we have the following,

$$\begin{aligned} & x + 12 \geq 0 \text{ and } x - 1 \geq 0 \text{ or } x + 12 \leq 0 \text{ and } x - 1 \leq 0 \\ \text{which implies} \quad & x \geq -12 \text{ and } x \geq 1 \text{ or } x \leq -12 \text{ or } x \leq 1. \end{aligned}$$

Theh solution to the inequatliy is two:
the first solution is,

$$x \geq -12 \text{ and } x \geq 1$$

or the second solution,

$$x \leq -12 \text{ and } x \leq 1.$$

Exercises

Solve the following inequalities.

1. $x^3 - 3x^2 + 3x - 1 \leq 0$
2. $x^2 - 8x + 7 \geq 0$
3. $0 \geq x^2 - a^2$
4. $0 \leq x^3 + 2x^2 - x - 2$
5. $5x^2 + 13x + 6 \leq 0$
6. $9x^2 + 10x + 1 \geq 0$
7. $-x^2 + 2x + 3 \geq 0$
8. $4x + 3 \leq -x^2$
9. $x^2 \geq 1$
10. $6x^2 \leq 2x + 28$