Sine Law
The Ambiguous Case

Raise My MArks

Raisemymarks.com

2020

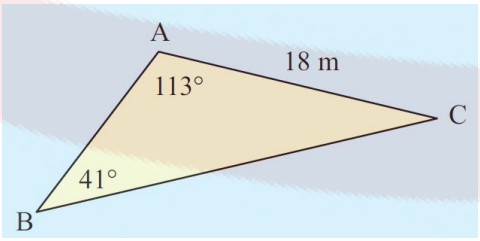
Sine Law: Ambiguous Case

Sine Law

Recall the sine law: Given a ΔABC we have the following relationship between the angles and sides,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Let's consider an example where we are looking for a missing side. For the triangle below, what is the length of side $a = BC$?



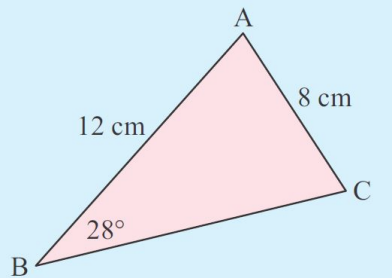
Solution By the sine law we have

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 113°}{a} &= \frac{\sin 41°}{18} \\ \frac{18(\sin 113°)}{\sin 41°} &= a, \quad \text{using cross multiplication} \\ \therefore 25.26cm &= a \end{aligned}$$

When looking for an angle using the Sine Law there are two possible answer that can result, an acute angle and an obtuse angle. This is what is referred to as the *ambiguous case*. Let's look at the following example.

Ambiguous Case

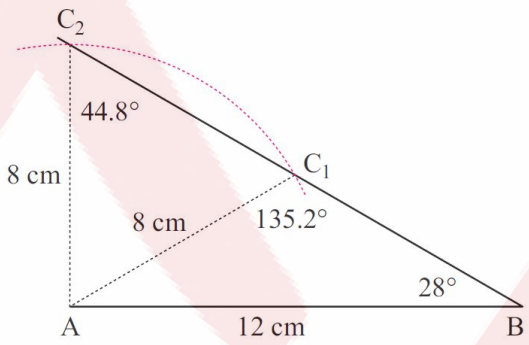
For the triangle below, find the measure of $\angle C$ in ΔABC if $AC = 8cm$, $AB = 12cm$ and $\angle B = 28°$.



Solution Using the Sine Law we have,

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin B}{b} \\ \frac{\sin C}{12} &= \frac{\sin 28^\circ}{8} \\ \sin C &= \frac{12(\sin 28^\circ)}{8} \\ \therefore \angle C &\approx 44.8^\circ \end{aligned}$$

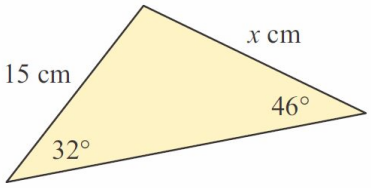
But $\angle C$ can also be obtuse or $\angle C = 180 - 44.8^\circ = 135.2^\circ$. There isn't enough information to eliminate one of the two angles so both must be included as possible answers. The diagram below shows how it can be possible to have a triangle with either the acute or obtuse angle measures.



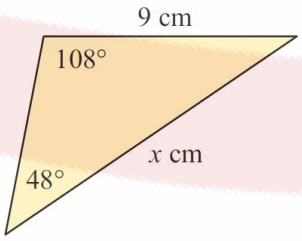
Exercises

1. Find the value of x for the triangles below.

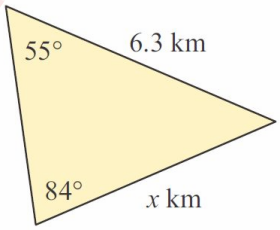
a)



b)



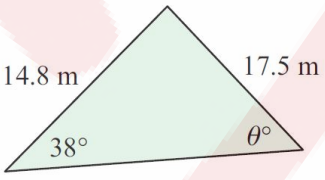
c)



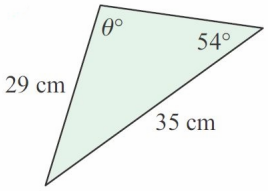
2. In ΔABC find,
   a) a if $A = 65°$, $B = 35^{c}irc$ and $b = 18cm$
   b) b if $A = 72°$, $C = 27°$ and $c = 24cm$
   c) c if $B = 25°$, $C = 42°$ and $a = 7.2cm$.

3. Find the value(s) of θ for the triangles below.

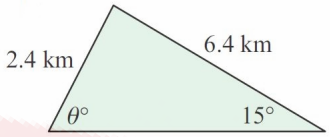
a)



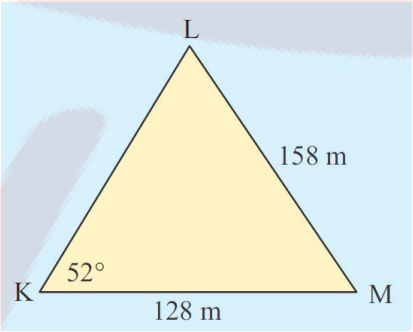
b)



c)



4. Find the measure of $\angle L$ for the triangle below,



5. In ΔABC find the measure of

   a) $\angle A$ if $a = 12.6cm$, $b = 15.1cm$ and $\angle B = 65°$

   b) $\angle B$ if $b = 38.4cm$, $c = 27.6cm$ and $\angle C = 43°$

   c) $\angle C$ if $a = 5.5km$, $c = 4.1km$ and $\angle A = 71°$.