

Scalar equation of a line in \mathbb{R}^2

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Scalar or Cartesian equation of a line in \mathbb{R}^2

If we are given a vector $\underline{n} = (A, B)$ perpendicular to the line l then the scalar or Cartesian equation of the line is given by

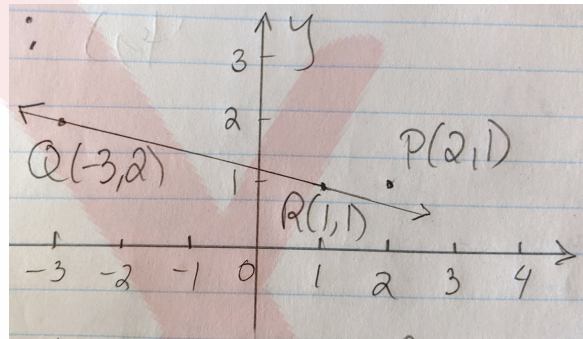
$$Ax + By + C = 0$$

We can take a point $P(x, y)$ on the line and plug it into the line to find the value for C .

Examples

Find the scalar equation of the line through P and in the direction of QR where $P=(2, 1)$, $Q = (-3, 2)$, $R=(1, 1)$.

Solution:



Let's find the slope of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{2 - 1}{-3 - 1} = \frac{1}{-4} \quad (1)$$

So far the equation of our line is,

$$y = mx + b = -\frac{1}{4}x + b$$

We need to find the y-intercept b . We know the point $P=(2,1)$ lies on the line. So we can plug the point P into the equation (1) and then solve for b .

$$\begin{aligned} 1 &= -\frac{1}{4}(2) + b \\ 1 + \frac{1}{2} &= b \\ \frac{3}{2} &= b \\ \therefore y &= -\frac{1}{4}x + \frac{3}{2}. \end{aligned}$$

The scalar equation of the line is,

$$\begin{aligned} y &= -\frac{1}{4}x + \frac{3}{2} \\ 4y &= 4\left(-\frac{1}{4}\right)x + 4\left(\frac{3}{2}\right) \\ 4y &= -x + 6 \\ \therefore x + 4y - 6 &= 0 \end{aligned}$$

is the scalar equation of the line.

Exercises

Given the points P , Q and R below, find the scalar equation of the line through the point P and perpendicular to the direction QR .

a) $P(3, 2), Q(0, 2), R(-3, 1)$

i) $P(-3, -4), Q(2, 0), R(5, 4)$

b) $P(4, 1), Q(-1, 0), R(0, 4)$

j) $P(0, -4), Q(3, 0), R(-2, 3)$

e) $P(-5, 3), Q(0, 1), R(5, 4)$

k) $P(-2, 5), Q(2, 2), R(1, 3)$

f) $P(-2, 0), Q(1, 3), R(4, -1)$