Range of a Function

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What is the range of a function?

The range of a function is the set of all values the function f can take on. If we let our function values be $y = f(x)$, the range is all the values y values the function can take on.

Example

Let's consider an example. Find the range of the following function,

$$y = f(x) = 2x + 5$$

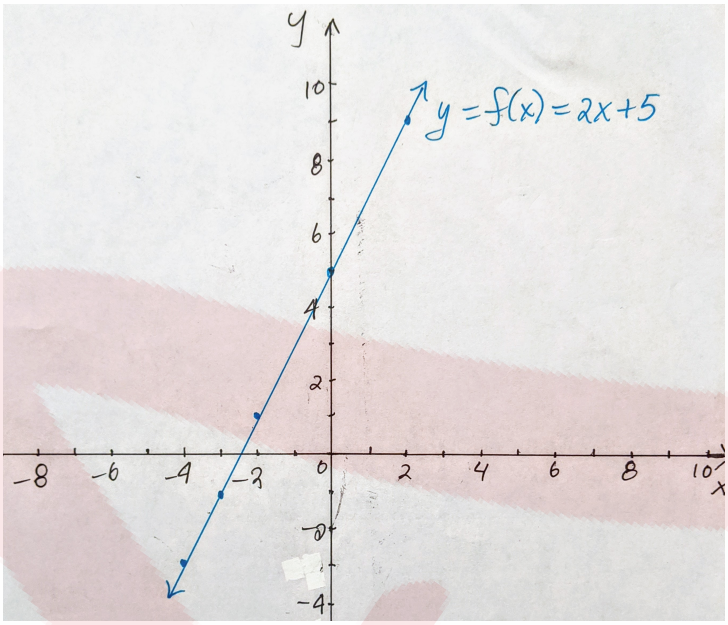
Solution Our domian is all real numbers, which means x can be any number. We want to determine what values y can take on. Let's create a table of values.

x	y = f(x)
0	5
$-\frac{5}{2}$	0
-2	1
2	9
-4	-3
-3	-1

We can see that as we put values for x into $y = f(x)$, y ranges through all real numbers. So, the range for f is all real numbers or,

$$Range f = \{y | y \in \mathbb{R}\}$$

We can also graph the function and visually determine the range, and domain.



From the graph we see that $y = f(x) = 2x + 5$ is a line that can take on any real value so any value on the y-axis. And, $f(x)$ can be evaluated at any value on the x-axis. So our range is the y-axis or $\mathbb{R}$ and our domain is the x-axis or $\mathbb{R}$.

Example

Let's consider the example of the function,

$$f(x) = x^2.$$

find the range.

Solution When we take the square of a number, can it ever be negative? No.

$$\begin{aligned}(-1)^2 &= 1\\ (-5)^2 &= 25\end{aligned}$$

This illustrates that our function is not defined for negative values of x. The proof of this will not be shown in this document. This implies, that the range is any real number greater than or equal to 0.

$$Range f = \{y \in \mathbb{R} | y \geq 0\}$$

Example

Let's consider another different example. What is the range of the following function?

$$f(x) = \frac{1}{x+3}$$

Solution Is there any value $f(x)$ cannot be? In this case, $f(x)$ is a fraction and a fraction can only equal 0 if the numberator is 0. In this case the numerator is 1. So, $f(x)$ can never be equal to 0. But, $f(x)$ can be any other nuegative or positive value.

$$Range f = \{y \in \mathbb{R} | y \neq 0\}$$

Exercises

What is the range of the following functions?

a) $3x - 7$

b) $x^2 + 2$

c) $-x^2$

d) $\frac{1}{x+5}$

e) $\sqrt{x+5}$

f) $\frac{1}{\sqrt{x^2-16}}$