Quadratics and Factoring
"Nice" Quadratics

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Quadratics

A quadratic is a polynomial with degree 2. Some examples of quadratics are,

$$x^2,\ 34y^2 - 24y + 14,\ x^2 - 1,\ y^2 + 2y + 1$$

Notice that a perfect square and a difference of squares are quadratics as well.

What is the form of a quadratics?

The general form of a quadratics is,

$$x^2 + bx + c \text{ or } ax^2 + bx + c \tag{1}$$

What is the factorization of a quadratic?

The factorization of a quadratic isn't as straight forward as the factorization for a perfect square or difference of square. A quadratic can either be factored "nicely" or we have to use the *quadratic formula*, or it may not be possible to factor at all, within the real numbers but that's something we won't get into here.

Note: Every polynomial can be factored over the complex numbers, $\mathbb{C}$. However, unless explicitly stated, we will assume we are working in the real numbers, $\mathbb{R}$.

The factorization for a "nice" quadratic

The factorization for a "nice" quadratic is given by,

$$abx^2 + (as + rb)x + rs = (ax + r)(bx + s) \tag{2}$$

where a, b, r, s are constant numbers and x is the variable.

Steps for factoring "nice" quadratics

Let's consider a few examples of a quadratics that factor "nicely" or " nice" quadratics.

Example Factor the following quadratic,

$$x^2 + 5x + 6$$

1. **What are the factors of the constant term?** What are the factors of 6? The factors of 6 are, 1 and 6 or 2 and 3 or their negatives. e.g. -1 and -6 or -2 and -3.

2. **What are the factors of the coefficient of the quadratics term?** The *quadratic term* is the term that has the degree 2 in it. So in our case x^2 is the quadratic term. The *coefficient* of the quadratic term, is the number in front of it. In this case there is no number which means the number is 1. What are the factors of 1? 1 and 1 are the factors of 1 or -1 and -1 are the factors of the 1.

3. **Do any products and sums of the factors in 1. and 2. give the coefficient of the linear term?** What is the *linear term*? The linear term is the term that has degree 1. So in our case $5x$ is the linear term. The coefficient of this linear term is the number in front of the x. So in our case the 5. So our question becomes, "Do any products and sums of the factors in 1. and 2. give the number 5?" Yes. 2 and 3 add up to 5. e.g. $2 + 3 = 5$.

4. **Write out the factorization for our quadratic.** This means we have,

$$x^2 + 5x + 6 = (x + 2)(x + 3) \tag{3}$$

5. **Let's check that this factorization is correct.** We need to multiply out the left had side of (3) and verify we obtain the right hand side,

$$\begin{aligned} (x + 2)(x + 3) &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

   Success! We have found the correct factorization.

What happens when the quadratic is not "nice"? You can find the answer to that question in the *Quadractic Formula* exercises sheet.

Exercises

For each of the following quadratics below answer the following:

a) Is the quadratic a perfect square? Justify.

b) Is the quadratic a difference of squares? Justify.

c) Can the quadratic be factored "nicely"? If "yes", factor the quadratic.

1. $x^2 + 3x + 2$
2. $-2x^2 - 11x + 6$
3. $-2x^2 + 4x + 6$
4. $3 + 2x - x^2$
5. $5x + 8x^2 + 3$
6. $x^2 + 9x + 20$