Quadratic Formula

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Quadratic

A quadratic is a polynomial with degree 2. Some examples of quadratics are,

$$x^2,\ 34y^2 - 24y + 14,\ x^2 - 1,\ y^2 + 2y + 1$$

Notice that a perfect square and a difference of squares are quadratics as well.

What is the form of a quadratic?

The general form of a quadratics is,

$$x^2 + bx + c \text{ or } ax^2 + bx + c \tag{1}$$

What is the factorization of a quadratic?

The factorization of a quadratic isn't as straight forward as the factorization for a perfect square or difference of square. A quadratic can either be factored "nicely" or we have to use the *quadratic formula*, or it may not be possible to factor at all, within the real numbers but that's something we won't get into here.

Note: Every polynomial can be factored over the complex numbers, $\mathbb{C}$. However, unless explicitly stated, we will assume we are working in the real numbers, $\mathbb{R}$.

We have looked at "nice quadratics" and how to factor those. Here we will look at factoring the not nice quadratics and how we factor those quadratics. This involved the **quadratic formula**.

Example

Let's consider another example. Factor the following quadratic,

$$x^2 + 4x + 2$$

Solution We start by going through the steps used for "nice" quadratics.

1. **What are the factors of the constant term?** We see that it is not possible to find factors of 2 that add up to 4.

In this case we need to use something called the *quadratic formula*. For a general quadratic,

$$ax^2 + bx + c \tag{2}$$

the quadratic formula gives us two values x_1 and x_2 so that we can factor (2)

$$ax^2 + bx + c = (x - x_0)(x - x_1) \tag{3}$$

What is the quadratic formula?

The **quadratic formula** is a formula that gives the numbers of x_0 and x_1 in equation (3) and allows us to factor a general quadratic given in (1). The quadratic formula is given by,

$$x_0, x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

provided,

$$b^2 - 4ac \geq 0. \tag{5}$$

The left hand side of equation (5) is usually referred to as the **discriminant**. The discriminant is used to determine if a quadratic can be factored at all. If the discriminant is greater than or equat to 0 then, yes the quadratic can be factored. Otherwise, it cannot be factors.

For our particular example,

$$\begin{aligned}
x_0, x_1 &= \frac{-4 \pm \sqrt{16 - 4(1)(2)}}{2} \\
&= \frac{-4 \pm \sqrt{16 - 8}}{2} \\
&= \frac{-4 \pm \sqrt{8}}{2} \\
&= \frac{-4 \pm \sqrt{(4)(2)}}{2} \\
&= \frac{-4 \pm \sqrt{4}\sqrt{2}}{2} \\
&= -2 \pm \frac{2\sqrt{2}}{2} \\
x_0, x_1 &= -2 \pm \sqrt{2} \text{or } x_0 = -2 + \sqrt{2} \text{ and } x_1 = -2 - \sqrt{2}.
\end{aligned}$$

Therefore we have,

$$\begin{aligned}
& (x^2 + 4x + 2) \\
&= (x - (-2 + \sqrt{2}))(x - (-2 - \sqrt{2})) \\
\therefore (x^2 + 4x + 2) &= (x + 2 - \sqrt{2})(x + 2 + \sqrt{2})).
\end{aligned}$$

We can verify that this correct by muliplying out the left hand side of the third line and showing it equals the right hand side.

Exercises

For each of the following quadratics below answer the following:

a) Can the quadratic be factored? Justify.

b) Is the quadratic a "nice" quadratic? Justify?

c) If the quadratic can be factored, factor.

1. $x^2 + 3x + 1$
2. $-2x^2 - 4x + 6$
3. $2x^2 + 4x + 6$
4. $x^2 - 3x - 2$
5. $3 + 2x + x^2$
6. $5x + x^2 + 3$
7. $-3 + 2x + x^2$
8. $x^2 + 9x + 7$