Powers

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Powers

A **power** is a number or variable or even a plynomial rasied to anothe rnumber or exponent or power. For example,

$$3^2, x^3, (x-9)^2$$

where $2, 3, 2$ are the powers or exponents of each term, respectively. There are two parts to a "power". There is the **base** and there is the **exponent**. In the examples above, $3, x$ and $(x-9)$ are the **bases**, respectively and $2, 3$ and 2 are the **exponents**, respectively.
Let's consider multiplying, dividing and powers of powers. When multiplying or dividing two powers with different bases, we do nothing.

$$3^2 4^3,\ x^2 6^3,\ 2^3 x^{-4}$$
$$\frac{5^2}{3^3},\ \frac{x^6}{9^2},\ \frac{4^{-2}}{x^3}$$

We cannot simplify any of the expressions above. If we have the same base when multiplying or dividing then we have rules for simplifying expressions. For example, if we consider

$$3^2 3^3 = 3^{2+3} = 3^5$$

we add the exponents. If we divide two powers with the same base,

$$\frac{4^4}{4^3} = 4^{4-3} = 4^1$$

we subtract the exponents. Our general rules for powers are,

Power rules

Multiplying powers

When multiplying two or more powers with the same nase, add the exponents.

$$a^m a^n = a^{m+n}$$

Dividing Powers

When dividing two powers with the same base, subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$$

Power of a power

Let's consider the power of a power. For example,

$$(4^2)^3,\ (x^2)^2,\ (3x^2)^3.$$

In this situation we are multiplying the powers together.

$$4^12,\ x^4,\ 3^2(x^2)^3 = 27x^6$$

Notice in the third example we did each facto sepaately then multiplied them both together.

$$(3x^2)^3 = (3^3)(x^2)^3 = 27x^6$$

The rule for powers of powers is, When taking a power of a power, multiply the exponents together.

$$(a^m)^n = a^{mn}$$

Rules for Powers

Multiplication

$$a^m a^n = a^{m+n}$$

Division

$$\frac{a^m}{a^n} = a^{m-n}$$

Powers

$$(a^m)^n = a^m n$$

Adding and Subtracting powers

Adding and subtracting powers is only really possible when the powers are exactly the same.

$$3^2 + 3^2,\ 4^3 - 4^3 + 24^3$$
$$x^2 - 5x^2,\ -2x + 5x + 3x$$

In this case we would have,

$$\begin{aligned} 3^2 + 3^2 &= 2(3^2) \\ 4^3 - 4^3 + 2(4^3) &= 2(4^5) \\ -2x + 5x + 3x &= 6x \end{aligned}$$

Back to polynomials. Now that we know how to manipulate powers, let's apply arithmetic operations to polynomials.

Adding Polynomials

When adding any number of polynomials, we add like terms. For example,

$$\begin{aligned} & (2x^2+3x-4)+(-5x+x^2+1), && \text{group like terms} \\ = \; & (2x^2+x^2)+(3x-5x)+(-4+1), && \text{add or subtract like terms} \\ = \; & 3x^2+(-2x)+(-3) \\ = \; & 3x^2-2x-3 \end{aligned}$$

Subtracting Polynomials

When subtracting polynomials it is exaly like adding except we subtract.

$$\begin{aligned} & (-4x^2+2x-3)-(5x^2+3x-9) \\ = \; & (-4x^2-5x^2)+(2x-3x)+(-3-(-9)) \\ = \; & -9x^2+(-x)+(-3+9) \\ = \; & -9x^2-x+6 \end{aligned}$$

Exercises

1. Simplify the following powers.

a) $(4^2)(4^3)$

b) $(a^4)(a^3)(a^2)$

c) $\frac{b^6}{b^2}$

d) $\frac{5x^2}{x^3}$

e) $\frac{a^2a^3}{a^4}$

f) $(4^3)^2$

g) $(3^2)^3$

h) $\frac{(3^2)(3^5)}{(3^3)^2}$

i) $\frac{b^2a^3(c^4)^2}{(ab)^3(ca)^2}$

j) $\frac{(4x)^2}{(2x)^3}$

k) $\frac{(2a)^2(3x^2)^3}{(3x^2)^3(4z)^2}$

l) $(bc)^3(ab^2c)^2$

m) $(x^2y)^3(2x)^2$

2. Add or subtract the following polynomials.

a) $(3x^3 + 4x^2 - 2x) + (x^2 - 5x + 8)$

b) $(-6x^2 + 7) + (14x - 9 + x^2)$

c)