Point of Intersection

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Point of Intersection of two lines

When we are in the Cartesian plane or $\mathbb{R}^2$ and we have two lines they are either *parallel* or eventually *intersect*. How do we find this point of intersection? Let's consider the following two lines,

$$y = -3x - 2 \text{ and } 2x + 3y = 5$$

How do we figure out the point of intersection? First we need to determine if the lines are parallel or not. Our two lines are,

$$\begin{aligned} l_1 &: y = -3x - 2 \\ l_2 &: 2x + 3y = 5 \end{aligned}$$

Rewrite all equations in the form $y = mx + b$.

$$\begin{aligned} l_1 : y &= -3x - 2 \\ l_2 : 2x + 3y &= 5 \\ 3y &= 5 - 2x \\ 3y &= -2x + 5 \\ y &= -\frac{2}{3}x + \frac{5}{3} \end{aligned}$$

Next, compare slopes. If the slopes of the two lines l_1 and l_2 are equal for both lines, then the lines are *parallel*. In our case,

$$m_1 = 3 \neq m_2 = -\frac{2}{3}$$

So, l_1 and l_2 are not parallel. Next we need to find a point (x_0, y_0) that is on the line l_1 and l_2. So, we need to solve for x_0 and y_0 using l_1 and l_2. How do we do this?

$$\begin{aligned} l_1 : y &= -3x - 2 \\ l_2 : y &= -\frac{2}{3}x + \frac{5}{3} \end{aligned}$$

Since $y = -3x - 2$ we plug y into l_2.

$$\begin{aligned}
-3x - 2 &= -\frac{2}{3}x + \frac{5}{3} \\
-2 - \frac{5}{3} &= -\frac{2}{3}x + 3x, \text{ all } x\text{'s on one side, numbers on the other side of } = \\
-\frac{6}{3} - \frac{5}{3} &= -\frac{2}{3}x + \frac{9}{3}x \\
-\frac{11}{3} &= -\frac{7}{3}x \\
-3\left(\frac{11}{3}\right) &= -3\left(\frac{7}{3}\right)x, \text{ divide both sides by 3} \\
\frac{-11}{-1} &= \frac{-7x}{-1}, \text{ divide both sides by } -1 \\
11 &= 7x, \text{ divide both sides by 7} \\
\frac{11}{7} &= x
\end{aligned}$$

So we now have,

$$x_0 = \frac{11}{7}.$$

To find y_0, plug x_0 into l_1 or l_2.

$$\begin{aligned}
y_0 &= -3x_0 - 2 \\
&= -3\left(\frac{11}{7}\right) - 2 \\
&= -\frac{33}{7} - 2 \\
&= -\frac{33}{7} - \frac{14}{7} \\
y_0 &= -\frac{47}{7}
\end{aligned}$$

Therefore, the point of intersection of l_1 and l_2 is,

$$(x_0, y_0) = \left(\frac{11}{7}, -\frac{47}{7}\right).$$

Exercises

For the pairs of lines, determine if they are parallel. If not, determine the point of intersection.

a) $y = 3x - 3$, $y = \frac{1}{3}x + 4$

b) $y = 2x + 4$, $y = -2x - 3$

c) $y = 5x + 1$, $y = 2x + 1$

d) $y = \frac{7}{2}x - 2$, $y = -\frac{2}{7} - \frac{1}{7}$

e) $y = -x + 1$, $y = x + 2$