Perfect Square

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2020

Perfect Square

We're going to continue looking at factoring polynomials but now we're considering the very *special* polynomial called a **perfect square**.

What is a perfect square?

A **perfect square** is a polynomial with a very specific format. This format is below,

$$a^2 + 2ab + b^2$$

where a and b are our two "variables". I put "variables" in quotes because $a = 5x$ or $b = \frac{1}{y}$. a and b can be anything, a term, a number, a single variable or even another polynomial. Below are some examples of a perfect square polynomial.

$$x^2 + 2ax + a^2,\ x^2 - 2ax + a^2,\ b^2x^2 + 2abx + a^2b^2$$

Notice that there are two terms that are the square of some other term. So in the first example x^2 is the square of x and a^2 is the square of a. The is a third term that is the twice the product of the other two terms. So in this first example the term $2ax$ is 2 times xa where we have identified x and a as the terms that have been squared. It does take practice to recognize when a polynomial is a perfect square and what the "squared" terms are.

What is the factorization of a perfect square?

Good question. Once we know a polynomial is a perfect square we have the following factorization for the perfect square,

$$x^2 + 2ax + a^2 = (x + a)^2 \tag{1}$$

Let's verify that this is correct. We verify a factorization by multiplying it out and seeing if we get the original polynomial. Our original polynomial is $x^2 + 2ax + a^2$. Let's multiply out the factorization $(x + a)^2$ and see what we get.

$$\begin{aligned} (x + a)^2 &= (x + a)(x + a) \\ &= x^2 + xa + ax + a^2 \\ &= x^2 + ax + ax + a^2 \\ &= x^2 + 2ax + a^2 \end{aligned}$$

Success! We have obtained our orginial polynomial so the factorization of $x^2+2ax+a^2$ is $(x + a)^2$.

Now let's look at how we approach factoring a perfect square. We'll consider an example and the steps we go through when factoring a perfect square.

Example

Steps for factoring a perfect square

Factor the following polynomial.

$$b^2 + 10b + 25 \tag{2}$$

1. **Is there a common factor for each term?** No
2. **Is this a perfect square?** Yes. The two "squared" terms are b and 5 because $b^2 = b \times b$ and $25 = 5^2$. And the third and remaining term $10b$ is equal to $2 \times 5b$, which is twice the product of the other two terms, 5 and b.
3. **Apply the factorization for a perfect square to our case.** We want to apply the factorization in (1) to our case in (2). Let's let $x = b$ and $a = 5$. Now we have as our factorization for (2),

$$b^2 + 10b + 25 = (b + 5)^2$$

4. **Check that this factorization is correct.** We need to multiply out the factorization in step 3. to verify that we obtain our original polynomial in (2). We can check to see if this factorization works.

$$\begin{aligned} (b+5)^2 &= (b+5)(b+5) \\ &= b^2 + 5b + 5b + 25 \\ &= b^2 + 10b + 25 \end{aligned}$$

Success! We have factored correctly.

Below are some general forms perfect squares. a and b are constants and x is the variable.

$$\begin{aligned} x^2 + 2ax + a^2 &= (x+a)^2 \\ b^2x^2 + 2abx + a^2 &= (bx+a)^2 \\ x^2 - 2ax + a^2 &= (x-a)^2 \\ b^2x^2 - 2abx + a^2 &= (bx-a)^2 \end{aligned}$$

Exercises

1. Determine which of the following are perfect squares?

a) $x^2 - 81$

b) $121 + 44x + 4x^2$

c) $x^2 - 18x + 81$

d) $x^2 + 9x + 20$

e) $4x^2 - 36$

f) $6x^2 - 8x - 8$

g) $36 + y^2 + 12y$

h) $9x^2 - 24x + 4$

i) $7 - 6x - x^2$

j) $100 - x^2$

k) $a^2x^2 + b^2y^2 + 2abxy$

l) $16 - 8x - x^2$

2. For those polyonmials in 1. that are perfect squares, factor.