Parametric equation of a line.

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2020

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Recall the vector equation of a line l through the point $\underline{x}_0 = (x_{01}, x_{02})$ in the direction $\underline{m} = (m_1, m_2)$. Both $\underline{x}_0$ and $\underline{m}$ are vectors in $\mathbb{R}^2$. The vector equation of the line is then given by,

$$\underline{x} = \underline{x}_0 + \underline{m}t$$

where, t is any real number.

Parametric equation of a line in $\mathbb{R}^2$

Now, if we write out the individual components of the vector equation of the line, we will get the *parametric equation of a line*. So from the vector equation of a line in $\mathbb{R}^2$ above, we get the parametric equaiton of a line below:

$$\begin{aligned} x_1 &= x_{01} + tm_1 \quad (1) \\ x_2 &= x_{02} + tm_2 \quad (2) \end{aligned}$$

Equations 1 and 2 make up the parametric equation of a line through $\underline{x}_0$ in the direction of $\underline{m}$.

Parametric equation of a line in $\mathbb{R}^3$

We have a similar set of equations in $\mathbb{R}^3$. The parametric equation of a line in $\mathbb{R}^3$ passing through the point $\underline{x}_0$ and in the direction of $\underline{m} = (m_1, m_2, m_3)$ is given by,

$$\begin{aligned} x_1 &= x_{01} + tm_1 \quad (3) \\ x_2 &= x_{02} + tm_2 \quad (4) \\ x_3 &= x_{03} + tm_3 \quad (5) \end{aligned}$$

where equations 3, 4 and 5 make up the parametric equations of a line through $\underline{x}_0$ in the direction of $\underline{m}$.

Example in $\mathbb{R}^2$

Find the parametric equation of a line through P=(4, -5) in the direction of QR where Q = (1, 2) and R= (-3, 1).

Solution: First find the direction $\underline{m}$.

$$\begin{aligned} \underline{m} &= (m_1, m_2) = QR = R - Q \\ &= (-3, 1) - (1, 2) = (-4, -1) \end{aligned}$$

The vector equation of the line is,

$$(x, y) = (4, -5) + (-4, -1)t$$

where t is any real number or $t \in \mathbb{R}$. The parametric equation of the line is then given by,

$$\begin{aligned} x &= 4 - 4t \\ y &= -5 - t \end{aligned}$$

where t is any real number or $t \in \mathbb{R}$.

Example in $\mathbb{R}^3$

Find the parametric equation of the line through P in the direction of QR where P = (2, 0 , 1), Q=(3, -2, 1), R = (-3, 1, 2).

Solution: First, find the direction $\underline{m}$.

$$\underline{m} = R - Q = (-3, 1, 2) - (3, -2, 1) = (-6, 3, 1)$$

The vetor equation of the line is given by,

$$(x, y, z) = (2, 0, 1) + (-6, 3, 1)t, \qquad \text{where } t \text{ is a real number or } t \in \mathbb{R}.$$

The parametric equation of the line is given by,

$$\begin{aligned} x &= 2 - 6t \\ y &= 3t \\ z &= 1 + t \end{aligned}$$

where t is any real number or $t \in \mathbb{R}$.

Exercises

Given that a line passes through P and in the direction QR find the parametric equation of the lines below.

a) $P(3,2), Q(0,2), R(-3,1)$

b) $P(4,1), Q(-1,0), R(0,4)$

c) $P(-1,2,1), Q(2,0,-1), R(-1,3,0)$

d) $P(3,-2,0), Q(2,2,2), R(-1,2,-1)$

e) $P(-5,3), Q(0,1), R(5,4)$

f) $P(-2,0), Q(1,3), R(4,-1)$

g) $P(4,0,-3), Q(-1,2,0), R(1,1,1)$

h) $P(-1,-2,1), Q(4,3,-1), R(3,1,2)$

i) $P(-3,-4), Q(2,0), R(5,4)$

j) $P(0,-4,-1), Q(3,0,0), R(-2,3,1)$

k) $P(-2,5,-3), Q(2,2,-2), R(1,3,4)$