Orthocentre of a triangle

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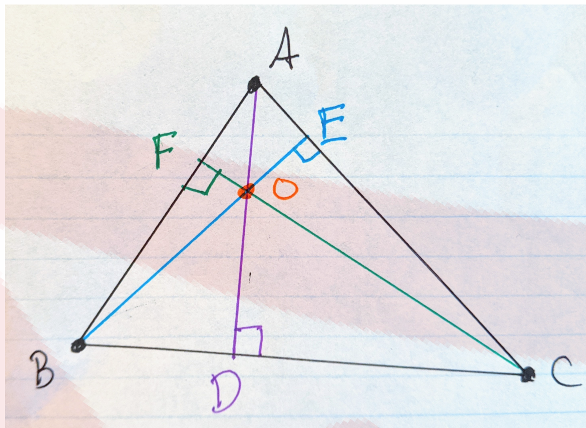
Orthocentre

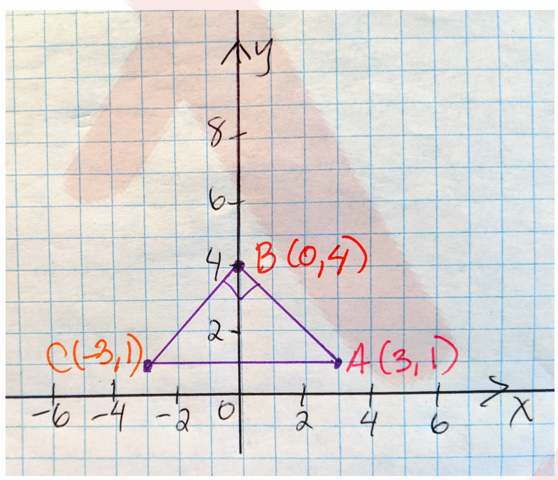
The *orthocentre* is the point of intersection of the three altitudes of a triangle. The *altitude* of a triangle is the line that goes through a vertex of a triangle and is perpendicular to the opposite side of the vertex.



In the diagram above, CF is the altitude of side AB; AD is the altitude of side BC; BE is the altitude of side AC.

Example: Find the orthocentre of the triangle with vertices B(0,4), A(3,1) and C(-3,1).

Solution: The first step is always to draw a diagram.



We know that the orthocentre is the point where the three altitudes of a triangle intersect. So, it is enough to find two of the altitudes of the triangle and then their point of intersection. We also know that the slope of two perpendicular lines are inverse reciprocals of each other. If we consider the alititude that goes through the vertex A(3,1), we know that this altitude has slope the negative reciprocal of the slope of the side BC of the triangle ΔABC. Let's find the slope of the line that goes through BC.

$$\begin{aligned} m_{BC} &= \text{slope of BC} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-4}{-3-0} \\ &= \frac{-3}{-3} \\ &= 1 \end{aligned}$$

Therefore, the slope of the altitude is,

$$m_{\perp BC} = -1.$$

The equation of the altitude is given by,

$$\begin{aligned} y &= m_{\perp BC}x + b \text{ or} && (1) \\ y - y_0 &= m_{\perp BC}(x - x_0) && (2) \end{aligned}$$

Let's use the second equation to find the equation of the altitude where $A(3,1) = (x_0, y_0)$.

$$\begin{aligned} y - y_0 &= m_{\perp BC}(x - x_0) \\ y - 1 &= -1(x-3) \\ y &= -x + 3 + 1 \end{aligned}$$

Therefore,

$$y = -x + 4$$

is the equation of the altitude to side BC through vertex A.

Now let's find the altitude to AB. We need to find the slope of AB.

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-1}{0-3} \\ &= \frac{3}{-3} \\ &= -1 \end{aligned}$$

Therefore, the slope of the altitude to AB is

$$m_{\perp AB} = 1$$

. The equation of the altitude to AB is,

$$y - y_0 = m_{\perp AB}(x - x_0)$$

where $C(-3,1) = (x_0, y_0)$.

$$\begin{aligned} y - y_0 &= m_{\perp AB}(x - x_0) \\ y - 1 &= 1(x+3) \\ y &= x + 3 + 1 \end{aligned}$$

Therefore,

$$y = x + 4$$

is the equation of the altitude to AB. Now we need to find the point of intersection of these two altitudes,

$$\begin{aligned} y &= -x + 4, \text{ equation of altitude to BC} \\ y &= x + 4, \text{ equation of altitude to AB.} \end{aligned}$$

To find the point of intersecvtion of two lines, let the y values for both lines be equal then solve for x. This will give the x coordinate of the point of intersection. We then take that x coordinate and plug it back into one of the two lines to find the y coordinate.

$$\begin{aligned} -x + 4 &= y = x + 4 \\ -x + 4 &= x + 4 \\ -x - x &= 4 - 4 \\ -2x &= 0 \\ x &= \frac{0}{-2} \\ x &= 0, \text{ is the x-coordinate of the point of intersection of the two altitudes.} \end{aligned}$$

The y coordinate is,

$$\begin{aligned} y &= -x + 4 \\ &= 0 + 4 \\ \therefore y &= 4 \text{ is the y-coordinate of the point of intersection of the two altitudes.} \end{aligned}$$

Thereforem the *orthocentre* of ΔABC is $(0, 4)$.

Exercises

Find the coordinates of the orthocentre of the triangle with the following vertices:

a) (2, -3), (8, -2), (8, 6)

b) (0, 2), (-2, 6), (4, 0)

c) (0, 0), (6, 0), (-2, 8)

d) (0, 6), (4, 6), (1, 3)

e) (3, 9), (1, 3), (10, 2)

f) (3, 1), (2, 2), (3, 5)