What is a linear system of equations?
Solving linear systems using substitution

What is a linear system of equations?

We know what a linear equation is. A linear system of equations i two or more linea equations grouped together. For example,

$$\begin{aligned} 2x - y + 6 &= 0 \\ -x + 4y - 7 &= 0 \end{aligned}$$

Usually when faced with a linear system of equations the goal is to solve it. How do you solve a linear system of equations? First, what does it mean to solve a linear system of equations? Solving a linear system of equations means finding values for x and y or a point $P(x, y)$ that lies on the lines in the linear system of equations. Now, how do we solve this linear system? There are two ways we'll consider,

1. Substitution

2. Elimination

Le'ts consider the above example of a linear system of equations,

$$\begin{aligned} l_1 : 2x - y + 6 &= 0 \\ l_2 - x + 4y - 7 &= 0 \end{aligned}$$

Substitution

Step 1: Using one of the two lines, isolate for one of the unknows, say y. So we'll use line l_1 or equation 1 and solve for y.

$$\begin{aligned} 2x - y + 6 &= 0 \\ 2x + 6 &= y \end{aligned}$$

Step 2: Take this expression for y and plug it into equation l_2.

$$\begin{aligned} -x + 4y - 7 &= 0 \\ -x + 4(2x + 6) - 7 &= 0 \end{aligned}$$

Step 3: Solve for the remaining variable, x in this case.

$$\begin{aligned} -x + 4(2x + 6) - 7 &= \\ -x + 8x + 24 - 7 &= 0 \\ 7x + 17 &= 0 \\ 7x &= -\frac{17}{7} \end{aligned}$$

Step 4: Use this value for x and plug it into the expression for y in Step 1.

$$\begin{aligned} y &= 2x + 6 \\ &= 2\left(-\frac{17}{7}\right) + 6 \\ &= -\frac{34}{7} + 6 \\ &= -\frac{34}{7} + \frac{42}{7} \\ y &= \frac{8}{7} \end{aligned}$$

The solution of the linear system of equations is therefore,

$$\left(-\frac{17}{7}, \frac{8}{7}\right)$$

Exercises

Solve the linear system of equation using substitution.

a)

$$
\begin{aligned}
3x - y &= 4 \\
y + 2x - 6 &= 0
\end{aligned}
$$

b)

$$
\begin{aligned}
2x + 4y - 6 &= 0 \\
-x + y + 1 &= 0
\end{aligned}
$$

c)

$$
\begin{aligned}
y &= 4x - 2 \\
5 &= 3x + 4y
\end{aligned}
$$

d)

$$
\begin{aligned}
7x + 9y + 10 &= 0 \\
-6x + 3y &= -4
\end{aligned}
$$