What is a linear system of equations?
Solving linear systems using elimination

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2020

What is a linear system of equations?

We know what a linear equation is. A linear system of equations i two or more linea equations grouped together. For example,

$$\begin{aligned} 2x - y + 6 &= 0 \\ -x + 4y - 7 &= 0 \end{aligned}$$

Usually when faced with a linear system of equations the goal is to solve it. How do you solve a linear system of equations? First, what does it mean to solve a linear system of equations? Solving a linear system of equations means finding values for x and y or a point $P(x, y)$ that lies on the lines in the linear system of equations. Now, how do we solve this linear system? There are two ways we'll consider,

1. Substitution
2. Elimination

Le'ts consider the above example of a linear system of equations,

$$\begin{aligned} l_1 : 2x - y + 6 &= 0 \\ l_2 - x + 4y - 7 &= 0 \end{aligned}$$

Elimination

The other method to solve a linear system of equations is elimination. Let's consider the same xample.

$$\begin{aligned} l_1 : 2x - y + 6 &= 0 \\ l_2 - x + 4y - 7 &= 0 \end{aligned}$$

Step 1: Choose which variable you want to eliminat first. Let's say x.
Step 2: Multiply each equation by a number so that the coefficint for x is the same in all equations. So let's multiple l_2 by 2. Now we have

$$\begin{aligned} l_1 : 2x - y + 6 &= 0 \\ l_2 - x + 8y - 14 &= 0 \end{aligned}$$

Step 3: Add or subtract the two equations to elimninate x.

$$\begin{array}{rr} & 2\text{x - y + 6} = 0 \\ + & \text{-2x + 8y - 14} = 0 \\ \hline & 0\text{x + 7y - 8} = 0 \end{array}$$

Step 4: Solve for the remaining variable, y in this case.

$$\begin{aligned} 7y - 8 &= 0 \\ 7y &= 8 \\ y &= \frac{8}{7} \end{aligned}$$

Step 5: We can do the same to find x or we can take the value for y that we found in Step 4 and substitute back into Step 1 and solve for x. Let's take the value for y and plug it back into one of the equations in Step 1. So we have $y = \frac{8}{7}$ and we will plug it into l_1.

$$\begin{aligned} l_1 : 2x - y + 6 &= 0 && \text{substitute } y = \frac{8}{7} \\ 2x - \frac{8}{7} + 6 &= 0 && \text{multiply through by 7} \\ 14x - 8 + 42 &= 0 && \text{now solve for } x \\ 14x + 36 &= 0 \\ 14x &= -36 \\ x &= \frac{-36}{14} \\ x &= -\frac{18}{17} \end{aligned}$$

Therefore, our solution is,

$$\left(-\frac{18}{7}, \frac{8}{7}\right)$$

Exercises

Solve the linear system of equations using elimination.

a)

$$\begin{aligned} 10x - 9y + 8 &= 0 \\ 5x + 4y - 7 &= 0 \end{aligned}$$

b)

$$\begin{aligned} 4x + 6y &= 5 \\ -3x + 2y &= 7 \end{aligned}$$

c)

$$\begin{aligned} 0 = &= 3x + 7 - 6 \\ 5 &= -2x + 5y + 1 \end{aligned}$$

d)

$$\begin{aligned} -2x &= 5 + 3y \\ 4y &= 6 - 2x \end{aligned}$$