

Interval of increase and/or decrease

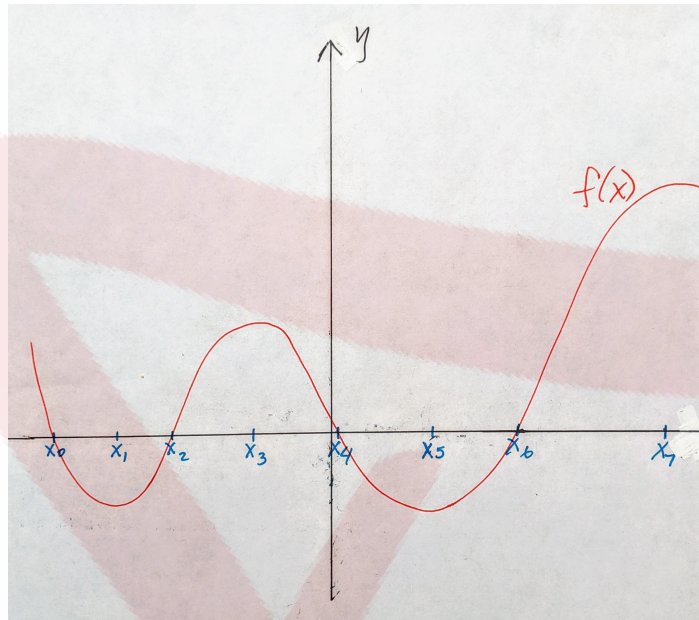
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2020

What is an interval of increase and/or decrease?

When we are given a function it is increasing and decreasing in value as we travel along the x-axis.



If we consider the example above of a function, we have some x-values of interest. First let's consider the x-values where the function crosses the x-axis. The values

$$x_0, x_2, x_4 \text{ and } x_6$$

are where the function f crosses the x-axis and are called the **roots** of the function f or the **x-intercepts** of the function f .

Let's consider the x-values where the function is either a maximum or a minimum. The values

$$x_1, x_3, x_5 \text{ and } x_7$$

are minimum and maximum values of the function f . These are also the x-values where the **vertices** of the function are located.

Now let's consider intervals. In particular the intervals where the function is positive or negative and intervals where the function is increasing or decreasing. First let's consider the intervals where the function is positive or negative. If we look at the graph of the function we see that the function is less than zero where

$$x_0 \leq x \leq x_2 \text{ or } x_4 \leq x \leq x_6.$$

The function f is greater than zero when,

$$x < x_0 \text{ or } x_2 < x < x_4 \text{ or } x > x_6.$$

This means, the intervals where the function is negative are

$$\begin{aligned} &x_0 \leq x \leq x_2 \text{ or } x_4 \leq x \leq x_6 \\ &[x_0, x_2] \text{ or } [x_4, x_6] \\ &\{x \in \mathbb{R} | x_0 \leq x \leq x_2 \text{ or } x_4 \leq x \leq x_6\} \end{aligned}$$

The intervals where the function is positive are,

$$\begin{aligned} &x < x_0, x_2 < x < x_4, x > x_6 \text{ or} \\ &(-\infty, x_0), (x_2, x_4), (x_6, \infty) \\ &\{x \in \mathbb{R} | x < x_0, x_2 < x < x_4, x_6 < x\} \end{aligned}$$

Now let's consider the **intervals of increase and decrease** of the function f . If we consider the function as traveling from left to right on a roller coaster, when are we going upwards and going downwards on the roller coaster will correspond to the intervals of increase and intervals of decrease of the function, respectively. Notice that the "direction" of increase or decrease of the function changes at the vertices of the functions. The vertices of the function above are

$$x_1, x_3, x_5 \text{ and } x_7.$$

These are the points where the direction of the function changes. The intervals where the function is decreasing are,

$$x \leq x_1, x_3 \leq x \leq x_5, x_7 \leq x.$$

The intervals where the function is increasing are,

$$x_1 < x < x_3, x_5 < x < x_7.$$

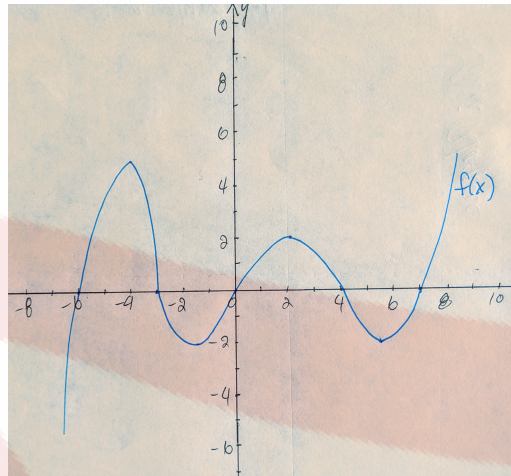
This means the intervals where the function decreases are,

$$\begin{aligned} &x \leq x_1, x_3 \leq x \leq x_5, x_7 \leq x \text{ or} \\ &(-\infty, x_1], [x_3, x_5], [x_7, \infty) \text{ or} \\ &\{x \in \mathbb{R} | x \leq x_1, x_3 \leq x \leq x_5, x_7 \leq \infty\} \end{aligned}$$

The intervals where the function increases are,

$$\begin{aligned} &x_1 < x < x_3, x_5 < x < x_7 \text{ or} \\ &(x_1, x_3), (x_5, x_7) \text{ or} \\ &\{x \in \mathbb{R} | x_1 < x < x_3, x_5 < x < x_7\}. \end{aligned}$$

Exercises



For the function f above,

1. On which intervals is the function positive?

2. On which intervals is the function negative?

3. On which intervals is the function increasing?

4. On which intervals is the function decreasing?

5. What are the roots of the function?

6. What are the vertices?

7. What are the maximum values?

8. What are the minimum values?