Graph Sketching

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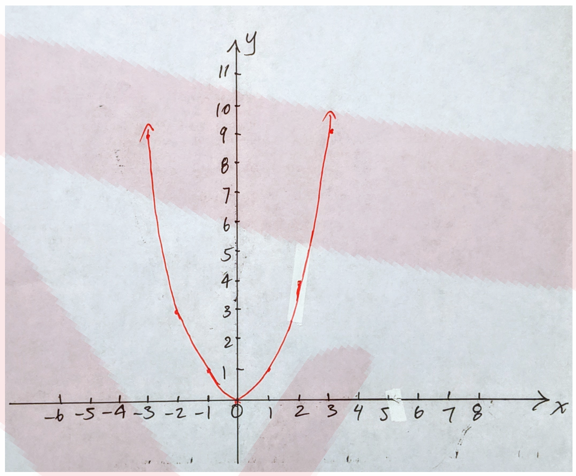
2020

Graph sketching

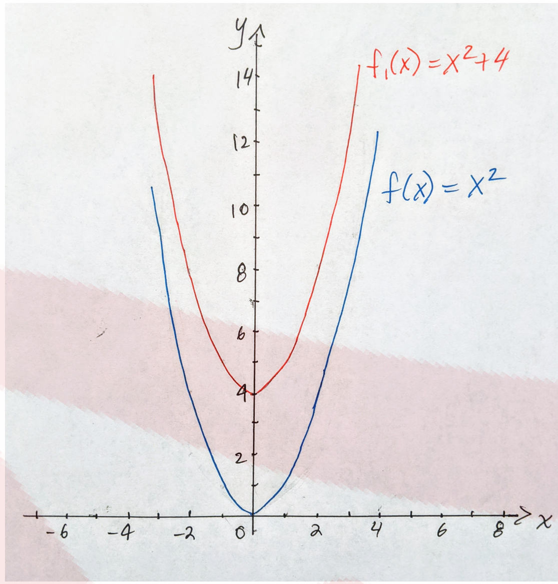
How do we determine the intervals of increase and decrease? We need to determine the vertices of the function. This brings us to sketching graphs and transformations of functions. We will consider sketching quadratics here. Let's consider the most basic quadratic, $f(x) = x^2$, and it's graph. Note, quadratics may also be called *parabolas*.



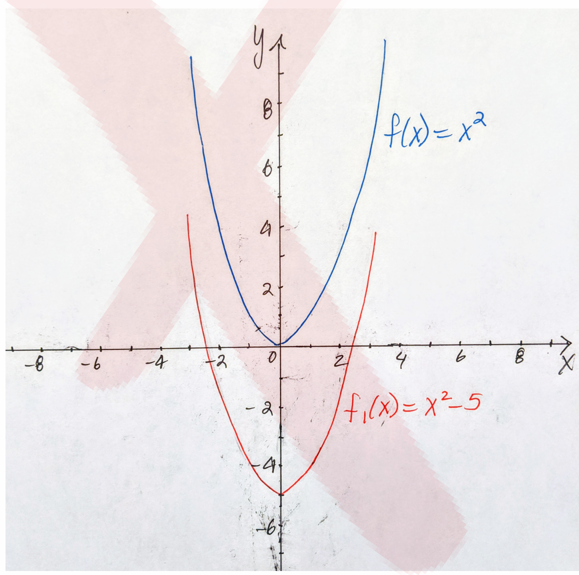
We will call this function a *parent function*. There are many other parent functions but we will start with this one to illustrate the basic transformations.

Translations

Vertical translation Below is a graph of a vertical translation up by 4 units of the parent function $f(x) = x^2$.



and below is a graph of a vertical translation down by 5 units.



A vertical translation moves the parent function $f(x) = x^2$ up or down along the y-axis. A general **vertical translation** is written as,

$$f(x) = x^2 + h,$$

where $h \in \mathbb{R}$.

When $h > 0$ then the vertical translation is **upwards** h units.

When $h < 0$ then the vertical translation is **downwards** h units.

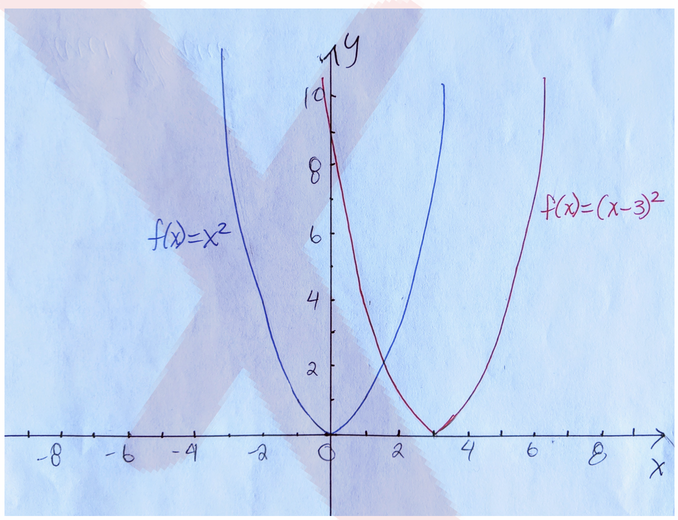
Horizontal translation A **horizontal translation** moves the parent function left or right along the x-axis. A general **horizontal translation** can be written as

$$f(x) = (x - k)^2,$$

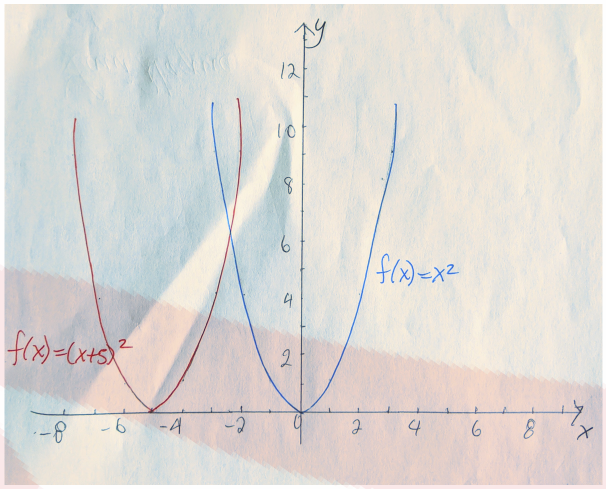
where $k \in \mathbb{R}$.

When $k > 0$ then the horizontal translation is to the **right** k units.

When $k < 0$ then the horizontal translation is to the **left** k units.
Below is a graph of a horizontal translation to the right 3 units

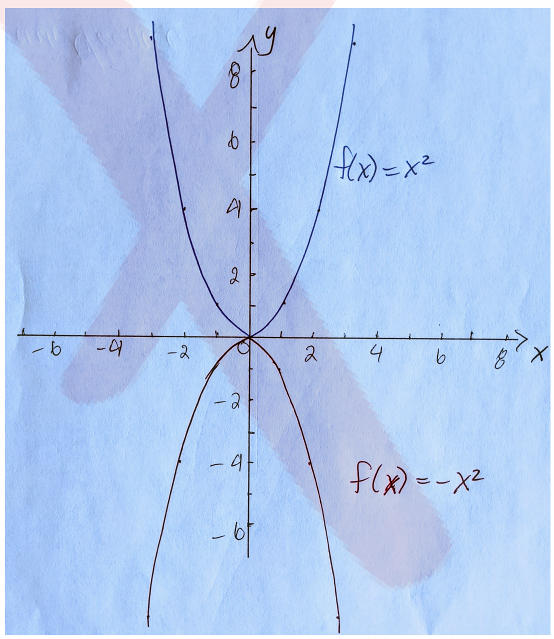


and below is a graph of a horizontal translation to the left 5 units.



Reflection

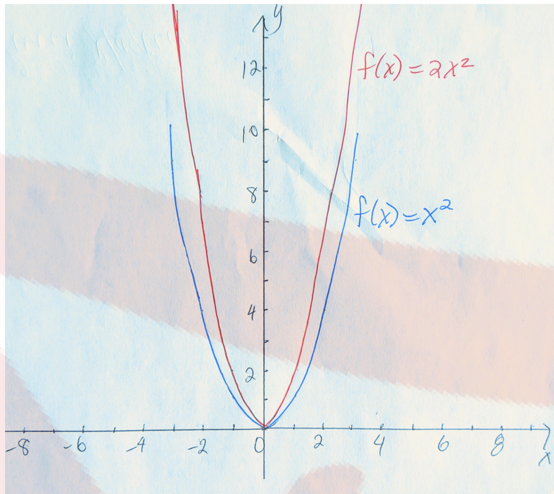
Reflection in the x-axis can be written as $f(x) = -x^2$.



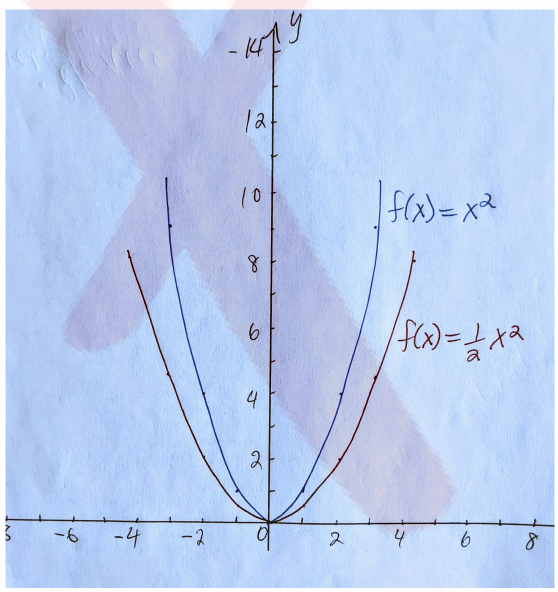
Reflection in the y-axis can be written as $f(x) = (-x)^2 =^2$.

Stretch or Compression

An example of a **vertical stretch** is the following function, $f(x) = 2x^2$.



An example of a **vertical compression** is the following function, $f(x) = \frac{1}{2}x^2$.



A vertical compression or stretch may be written as,

$$f(x) = ax^2,$$

where $a \in (0, \infty)$.

When $a > 1$ then we have a **vertical stretch**.

When $a < 1$ then we have a **vertical compression**.

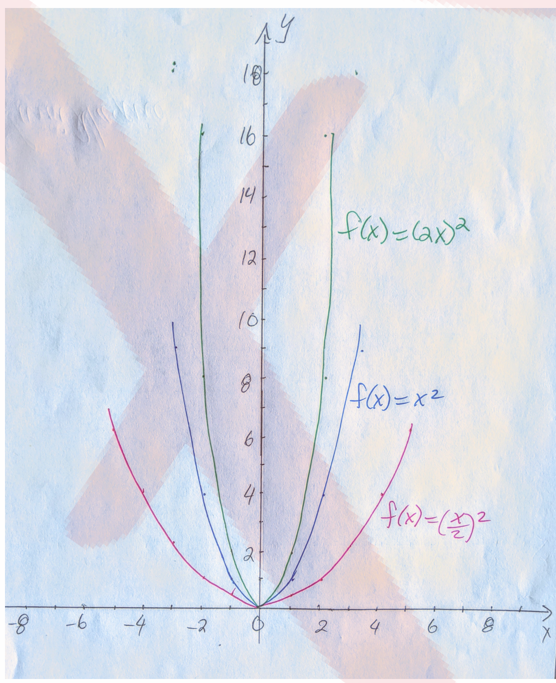
An example of a horizontal stretch or compression can be seen from the following function. This function represents a horizontal compression,

$$f(x) = (2x)^2 = 4x^2$$

is the same as a vertical stretch. And the following function represents a horizontal stretch,

$$f(x) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$$

which is the same as a vertical compression.



In general, a horizontal stretch or compression may be written as,

$$f(x) = (kx)^2,$$

where $k \in \mathbb{R}$.

When $k > 1$ then we have a **horizontal compression**.

when $k < 1$ then we have a **horizontal stretch**. Notice that horizontal compression is the same as a vertical stretch and a horizontal stretch is the same as a vertical compression.

Exercises

For each quadratic below, indicate if there is

a) a vertical translation, up or down

b) a horizontal translation, left or right

c) vertical stretch or compression and by how much

d) horizontal stretch or compression and by how much

e) reflection in the x-axis and/or y-axis

1. $y = x^2 + 3$
2. $y = x^2 - 7$
3. $y = 6x^2 - 1/2$
4. $(x - 3)^2 + 4$
5. $-(x + 2)^2 - 1$
6. $y = \frac{1}{2}(x - 1)^2 + 2$
7. $y = -x^2$
8. $y = (x + 6)^2 + 6$

9. $\frac{1}{3}(x-4)^2+1$

11. $y=(x+2)^2$

10. $-\frac{1}{4}x^2+10$

12. $(\frac{1}{2}x-4)^2-1$