Forms of a Quadratic & Solving Quadratics

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Forms of quadratics

There are three basic forms of quadratics.

1. **Standard or scalar form**

$$Ax^2 + Bx + C = y, \;\; A, B, C \in \mathbb{R} \tag{1}$$

2. **Vertex form**

$$a(bx - h)^2 + k = y, \;\; a, b, h, k \in \mathbb{R} \tag{2}$$

3. **Factored form**

$$(x - a)(x - b) = y, \;\; a, b \in \mathbb{R} \tag{3}$$

What does it mean to "solve" a quadratic?

Recall what a quadratic is. It is a polynomial of degree 2 and has the following general form,

$$ax^2 + bx + c \tag{4}$$

When you take this quadratic, or any polynomial, and let it equal 0, we now have an equation. "Solving" and equation means, finding the values of the variable(s) that allow the equation to be true or *satisfied*. So, "solve a quadratic" means, create the equation

$$ax^2 + bx + c = 0 \tag{5}$$

from (4) and find the values of x that *satisfy* this equation (5). Let's look at an example.

We will consider a standard or scalar form, equation (1), of a quadratic and "solve the quadratic".

Example

Solve the following quadratic, where $a, b, c \in \mathbb{R}$.

$$ax^2 + bx + c$$

Solution:

1. **Write the corresponding equation for the quadratic.** We do this by letting the quadratic equal 0.

$$ax^2 + bx + c = 0$$

2. **Can we factor the left hand side, or the non-zero side, of the equation?** This now becomes an exercise in facctoring a quadratic. So we need to go through the steps for factoring a quadratic. Determine if it is a perfect square; determine if it is a difference of squares. If it is not, determine if it can be factored at all using the *disciminant*. If it can be factored, then can it be factored "nicely"? If not, then use the quadratic formula. Recall the quadratic formula.

$$x_0, x_1 \quad = \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This means that

$$(x - x_0)(x - x_1) = ax^2 + bx + c = 0$$

and therefore, the solutions of

$$ax^2 + bx + c = 0.$$

are x_0 and x_1.

Exercises

1. Name the form of the quadratics below.

   a) $x^2 + 5x - 2$

   b) $5(x-2)^2 - 6$

   c) $-2(x - \frac{1}{2})(3x+1)$

   d) $9x^2$

   e) $-(2x+1)^2 + 3$

   f) $-6x^2 - 2x + 1$

   g) $2(x-3)(x+4)$

2. Solve the following quadratics, if possible. If not possible, explain why.

   a) $x^2 + 3x + 1$

   b) $-2x^2 - 4x + 6$

   c) $2x^2 + 4x + 6$

   d) $x^2 - 3x - 2$

e) $3 + 2x + x^2$

g) $-3 + 2x + x^2$

f) $5x + x^2 + 3$

h) $x^2 + 9x + 7$