Exponents

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Exponents

We have seen powers and somen of the rules for exponents. But, in these examples and rules the exponents have been positive integer values. Now we'll consider exponents that have the following types of values:

1. A value of 0
2. Negative numbers.
3. Rational Numbers (e.g. fractions)

Exponent value of 0

Let's consider an exponent value of 0.

$$4^0 = \text{the product of 4 zero times.} \ = 1$$

Actually the product of anything, variable or numeric, zero times is 1.

Rule for a 0 exponent

$$a^0 = 1, \quad \text{for any } a.$$

Negative exponents

Any time you see a negative exponent, the base and exponent move to the denominator. Let's consider some examples.

$$\begin{aligned} 5^{-1} &= \frac{1}{5^1} = \frac{1}{5} \\ 6^{-2} &= \frac{1}{6^2} \\ 3^{-3} &= \frac{1}{3^3} \\ 3x^{-2} &= \frac{3}{x^2} \end{aligned}$$

Rule for negative exponents

$$a^{-m} = \frac{1}{a^m}, \ m > 0.$$

Rational exponents

A rational number is a number that can be written a a fraction. For example,

$$\frac{1}{2}, \frac{3}{4}, \frac{7}{3}, -\frac{9}{4}, \frac{2}{3}, \text{ etc.}$$

All fractions can be written as decimals but not all decimals can be written as fractions. Let's consider an example with rational exponents.

$$\begin{aligned} 2^{\frac{1}{2}} &= \sqrt{2} \\ 6^{\frac{1}{3}} &= \sqrt[3]{6} \\ 8^{\frac{1}{3}} &= \sqrt[3]{8} \\ 36^{\frac{1}{2}} &= \sqrt{36} \\ 81^{\frac{1}{4}} &= \sqrt[4]{81} \end{aligned}$$

Rule for rational exponents of the form $\frac{1}{m}$

$$a^{\frac{1}{n}} = \sqrt[n]{a},\ n > 0.$$

Let's consider a rational number of the form $\frac{n}{m}$ now with an example.

$$\begin{aligned} 2^{\frac{3}{2}} &= \sqrt[2]{2^3} = \sqrt{2^3} \\ 8^{\frac{4}{3}} &= \sqrt[3]{8^4} = \sqrt[3]{4096} = 16 \text{ or} \\ 8^{\frac{4}{3}} &= (\sqrt[3]{8})^4 = 2^4 = 16 \\ 27^{\frac{2}{3}} &= (\sqrt[3]{27})^2 = 3^2 = 9 \end{aligned}$$

Rule for rational exponents of the form $\frac{m}{n}$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Exercises

Simplify the following.

1. 4^0
2. 1089^0
3. $15^{\frac{1}{2}}$
4. $8^{-\frac{1}{3}}$
5. 16^2
6. $4^2 8^{-\frac{1}{3}}$
7. $27^{\frac{2}{3}} 3^2$
8. $x^3 x^{\frac{5}{4}} y^{-2}$
9. $\frac{y^{-\frac{2}{3}} x^{\frac{5}{4}}}{x^{-\frac{3}{4}} y^{\frac{1}{3}}}$
10. $\frac{a^0 b^3 a^4}{b^2 a^3}$