Equation of a Quadratic

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Factored form of a Quadratic

What is the factored form of a quadratic? Suppose we have a quadratic function,

$$ax^2 + bx + c \tag{1}$$

with roots r and s. The **factored form** of the equation of the quadratic in (1) is given by,

$$(x - r)(x - s) \tag{2}$$

How do we determine the **factored form** of the equation of a quadratic? Let's consider an example to see how to find the factored form.

Example

Find the factored form of the following quadratic.

$$x^2 + 5x + 6.$$

Solution What are the roots of the above quadratic? We notice that $x = -3$ aor when $x = -2$ then our fuction $x^2 + 5x + 6 = f(x)$ equals zero.

$$\begin{aligned} f(-3) &= (-3)^2 + 5(-3) + 6 = 0 \\ f(-2) &= (-2)^2 + 5(-2) + 6 = 0 \end{aligned}$$

So the factors of $x^2 + 5x + 6$ are $(x + 3)$ and $(x + 2)$. Therefore, the factored form of $x^2 + 5x + 6$ is,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

We can also work backwards if we are given the roots of a quadratic, r and s. We can find the **standard equation** of the quadratic as follows,

$$\begin{aligned} (x - r)(x - s) &= x^2 - sx - rx + rs \\ &= x^2 - (s + r)x + rs \end{aligned}$$

Therefore,

$$x^2 - (x + r)x + rs$$

is the **standard form** of the equation of a quadratic with roots r and s.
Let's consider an example.

Example

Given a quadratic has roots 7 and 2, what is the equation of the quadratic?

Solution

1. First, determine the factors of the quadratic. In this case they are,

$$(x-7) \text{ and } (x-2)$$

2. Second, take the product of the fators.

$$\begin{aligned}(x-7)(x-2) &= x^2-2x-7x+14\\ &= x^2-9x+14\end{aligned}$$

Therefore, $x^2-9x+14$ is the standard form the quadratic with roots 7 and 2.

Exercises

Given the following roots, find the **standard form** of the quadratic and the **factored form** of the quadratic.

1. $x_1 = 3/2,\ x_2 = -4$
2. $x_1 = 5,\ x_2 = -2/3$
3. $x_1 = -2,\ x_2 = 2$
4. $x_1 = 1,\ x_2 = 1/4$
5. $x_1 = 6,\ x_2 = -3$
6. $x_1 = -4,\ x_2 = 9$
7. $x_1 = 3,\ x_2 = 0$
8. $x_1 = 7,\ x_2 = -5/2$
9. $x_1 = -1/2,\ x_2 = 1/3$
10. $x_1 = 8,\ x_2 = -6$