Equation of a Line
Part 2
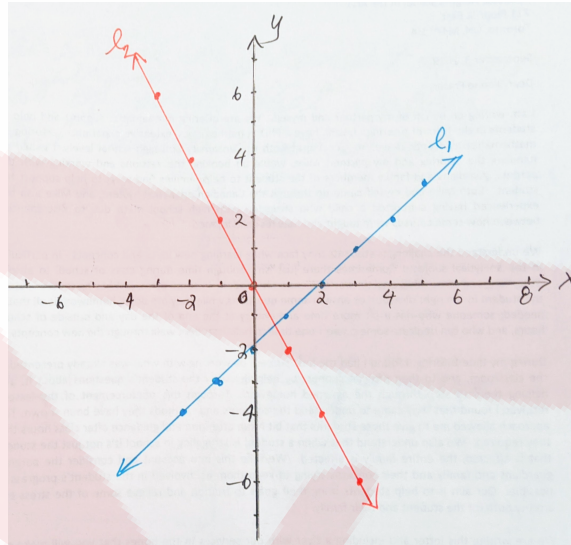
RaiseMyMarks.com

2020

Equation of a Line

A line can be thought of as the extension of a set of points that when joined create a straight line segment.

We have two lines l_1 and l_2. What are the slopes of l_1 and l_2, respectively? We need to find two points on l_1 and then calculate the *rise* and *run*. Two points on l_1: $(x_0, y_0) = (1, -1)$ and $(x_1, y_1) = (4, 2)$.

$$ slope = \frac{rise}{run} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{2 - (-1)}{4 - 1} = \frac{3}{3} = 1 $$

Therefore, the slope of line l_1 is 1. We usually represent the slope by m. So let's let the slope of l_1 be $m_1 = 1$.

Calculuate slope of l_2. Two points on l_2: $(x_0, y_0) = (-1, 2)$ and $(x_1, y_1) = (2, -4)$.

$$ slope \ of \ l_2 = m_2 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{-4 - 2}{2 - (-1)} = \frac{-6}{3} = -2 $$

Therefore, the slope of line l_2 is $m_2 = -2$.

The *equation of a line* in general is given by,

$$ y = mx + b $$

where m is the slope of the line and b is the y-interceiptof the line.

What is the y-interceipt?

The y-intercept is the point where the line crosses the y-axis. It casn also be calculated by sustituting $x = 0$ into the equation of a line. We can also find the y-interceipt by plugging the coordinates of a point on the line into the equation for the line.

Let's try and find the y-interceipt of l_1. We have the slope for l_1 as $m_1 = 1$. The equation of line l_1 so far is,

$$y = m_1 x + b_1 = x + b_1$$

where b_1 s the y-interceipt of l_1. Let's take a point on l_1 $(1, -1)$ and plug it into l_1.

$$\begin{aligned} y &= x + b_1 \\ -1 &= 1 + b_1 \\ -2 &= b_1 \end{aligned}$$

Now we have the y-intercept and the equation of the line l_1 is

$$\begin{aligned} l_1 : y &= m_1 x + b_1 \\ y &= x - 2 \end{aligned}$$

Let's find the complete equation for the line l_2. We already have that the slope is $m_2 = -2$. One point on line l_2 is $(x_0, y_0) = (-1, 2)$. Let's use this point to find the y-intercept.

$$\begin{aligned} y &= -2x + b_2 \\ 2 &= -2(-1) + b_2 \\ \therefore 0 &= b_2 \end{aligned}$$

Now we have the equation for line l_2 as,

$$\begin{aligned} l_2 : y &= m_2 x + b_2 \\ y &= -2x + 0 \\ y &= -2x \end{aligned}$$

Exercises

1. What is the slope of each line?

   a) $y = -2x$

   b) $y = x + 6$

   c) $y = -\frac{1}{2}x + 4$

   d) $y = \frac{5}{3}x - 5$

   e) $y = -7x + 10$

2. What is the y-interceipt for each line above?

3. Write the equation of the line with the following slope m and y-interceipt b.

   a) $m = -2, b = 0$

   b) $m = -\frac{1}{2}, b = 1$

   c) $m = 3, b = -5$

   d) $m = \frac{1}{4}, b = \frac{7}{2}$