Distance from a point to a line

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2020

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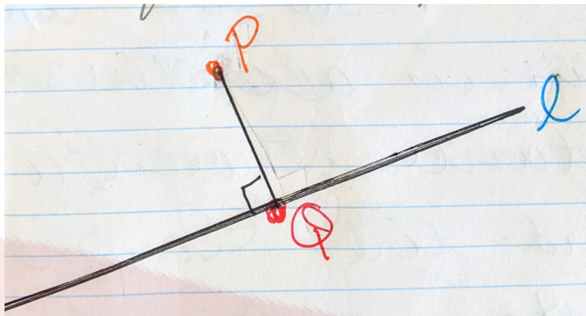


To find the distance of a point P to a line l we always consider the perpendicular distance from the point to the line. What does "perpendicular" distance mean? If we draw a line through the point P that intersects our line l at some other point Q, say, the distance from P to Q, PQ, is the "perpendicular" distance from the point P to l. This is also the shortest distance between a point and line. To find the distance the length or *distance formula* needs to be used. The distance formula is given by,

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{1}$$

where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, say. Let's consider an example.

Example

Find the shortest distance from P=(-1,3) to $x - y + 5 - 0$.

Solution

Step 1: We need to find the equation of the line through P and perpendicular to $x - y + 5 = 0$. We need to find the slope of the line.

$$\begin{aligned} x - y + 5 &= 0 \\ -y &= -5 - x \\ y &= x + 5 \end{aligned}$$

Therefore, the slope is $m = 1$. The slope of a line perpendicular to the given line $x - y + 5 = 0$ is $m_\perp = -\frac{1}{1} = -1$. Now, let's find the equation of the line through P with the slope $m_\perp = -1$.

$$\begin{aligned} y &= m_\perp x + b \\ y &= -x + b \end{aligned}$$

We need to find b. We know P lies on the line, so let's plug P=(-1, 3) in to find b.

$$\begin{aligned} 3 &= (-1)(-1) + b \\ 3 &= 1 + b \\ \therefore 2 &= b \end{aligned}$$

Therefore the equation of the line perpendicular to $x - y + 5 = 0$ through P = (-1,3) is $y = -x + 2$.

Step 2: We know $y = -x + 2$ is perpendicular to $x - y + 5 = 0$. Now we need to find the point of intersection, Q say, of these two lines. We find this point be equating the two lines,

$$\begin{aligned} y &= -x + 2 && (2) \\ x - y + 5 &= 0 \implies y = x + 5 && (3) \end{aligned}$$

Let's equation equations (2) and (3).

$$\begin{aligned} y = -x + 2 &= x + 5 \\ 2 - 5 &= x + x \\ -3 &= 2x \\ -\frac{3}{2} &= x \end{aligned}$$

Therefore, the x coordinate of the point of intersection is $-\frac{3}{2}$. Next, we need to find the y coordinate of the point of intersection. We do this by pluging the x coordinate into one of the equations (2) or (3). Let's plug the x coordinate into equation (2).

$$\begin{aligned} y &= -x + 2 \\ y &= -\left(-\frac{3}{2}\right) + 2 = \frac{3}{2} + 2 \\ y &= \frac{3}{2} + \frac{4}{2} \\ \therefore y &= \frac{7}{2} \end{aligned}$$

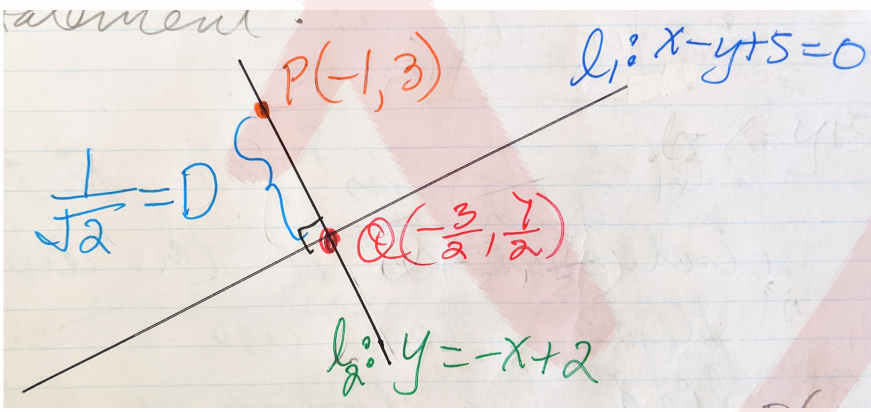
Therefore the y coordinate of the point of intersection is $y = \frac{7}{2}$. Therefore, the point of intersection is $Q = \left(-\frac{3}{2}, \frac{7}{2}\right)$.

Step 3: Now we need to find the distance between P = (-1, 3) and Q = $\left(-\frac{3}{2}, \frac{7}{2}\right)$. For this we need to use the distance formula (1).

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(-\frac{3}{2} + 1\right)^2 + \left(\frac{7}{2} - 3\right)^2} \\ &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{7}{2} - \frac{6}{2}\right)^2} \\ &= \sqrt{\frac{1}{2} + \left(\frac{1}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{1}{4}} \\ &= \sqrt{\frac{2}{4}} \\ \therefore D &= \sqrt{\frac{1}{2}} \end{aligned}$$

Step 4: We need a final statement.

Therefore, the distance between P=(-1, 3) and line $l: x - y + 5 = 0$ is $\frac{1}{\sqrt{2}}$.



Exercises

1. Find the shortest distance between the point P and the line l given below.

   a) $P = (2, 2)$; $l : y = x + 1$

   b) $P = (0, -1)$; $l : 5x + 2y + 3 = 0$

   c) $P = (-2, -1)$; $l : 2x + y + 3 = 0$

2. Find the distance from the piont A to the line joining B and C where,

   a) A = (-2, -2), B = (5, 2), C = (-1, 4)

   b) A = (5, 5), B = (2, -6), C = (5, 8)

   c) A = (1, 2), B = (2, -5), C = (6, -3)

3. A line has y-intercept Y and x-intercept X. Find the shortest distance from the point P to the line where X, Y and P are given below.

   a) X = (-6, 0), Y = (0, -3), P = (5, -11)

   b) X = (2, 0), Y = (0, -7), P = (9, 3)

   c) X = (7, 0), Y = (0, -4), P = (4, 10)