Difference of Squares

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Difference of Squares

We're going to continue looking at factoring polynomials but now we're considering the *special* polynomials that is called a **difference of squares**.

What is a difference of squares?

A **difference of squares** has the following structure:

$$b^2 - a^2$$

Notice that each term is a the square of some other term. So in this case b and a are the terms that are squared then subtracted, hence the name, "a differnce of squares". Below are some examples if a difference of squares.

$$x^2 - 100,\ 36a^2 - 81,\ x^2 - 16,\ 3x^2 - 1,\ y^2 - 1$$

What is the factorization of a difference of squares?

Good question. Once we know a polynomial is a difference of square, we have the following factorization,

$$x^2 - y^2 = (x - y)(x + y) \tag{1}$$

Let's verify that this factorization in (1) is correct by muliplying out the left hand side and verifying that we obtain the right hand side.

$$\begin{aligned}(x - y)(x + y) &= x^2 + xy - yx - y^2 \\ &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2\end{aligned}$$

Success! We obtained the right hand side of (1) so $(x - y)(x + y)$ is the factorization of $x^2 - y^2$.

Steps for factoring a difference of squares

Let's consider an example.

Example

Factor the following polynomial,

$$x^2 - 9 \tag{2}$$

Solution

1. **Is there a common factor for each term?** No
2. **Is this a difference of squares?** Yes. The two "squared" terms are x and 9 because $x^2 = x \times x$ and $9 = 3^2$ and they are being sutracted.
3. **Apply the factorization for a difference of squares.** We want to apply the factorization in (1) to our case in (2). Let's let $x = x$ and $a = 3$. Now we have as our factorization for (2),
$$x^2 - 9 = (x - 3)(x + 3)$$
4. **Check that this factorization is correct.** We need to multiply out the factorization in step 3. to verify that we obtain our original polynomial in (2). We can check to see if this factorization works.
$$\begin{aligned}(x - 3)(x + 3) &= x^2 - 3x + 3x - 9 \\ &= x^2 - 9\end{aligned}$$
Success! We have factored correctly.

Below is the most general form of a difference of squares. a and b are constants and x and y are variables.

$$a^2x^2 - b^2y^2 = (ax - by)(ax + by)$$

Exercises

1. Determine which of the following are a difference of squares?

a) $x^2 - 81$

b) $121 + 44x + 4x^2$

c) $x^2 - 81$

d) $x^2 + 9x + 20$

e) $4x^2 - 36$

f) $6x^2 - 8x - 8$

g) $36 - 9y$

h) $9x^2 - 24x + 4$

i) $7 - 6x - x^2$

j) $100 - x^2$

k) $a^2x^2 + b^2y^2 + 2abxy$

l) $16 - 8x - x^2$

2. Factor all those polynomials in 1. that are a difference of squares.