Composition of Functions
Identifying f and g

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What is the composition of functions?

The composition of functions means, one function is inserted into the another funcionn where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x, or $x = g(t)$ for $f(x)$. Let's a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the compostion of two plynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \textit{ and } g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution The compoistion of f and g looks like,

$$f \circ g(t) \textit{ or } f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t+1), \text{ where } x = g(t) = t+1 \\ &= (t+1)^2 + 3(t+1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.

Identifying f and g

Let's try another example.

Example

For the function

$$y = \frac{1}{2x+3},$$

determine f and g so that $y = f(g(x))$.

Solution: Looking at the function

$$y = \frac{1}{2x+3},$$

we want to try and find the "smallest" or most basic functions we can recognize. For example, a linear function x, or a quadratic x^2 or a rational function $\frac{1}{x}$ or a square root $\sqrt{x}$ etc. We can think of some of these "smallest" or "basic" functions as **parent functions**. Let's see if we can identify some parent functions in our given function. Immediately we see that we are dealing with a rational function or a fraction so $\frac{1}{x}$ is one possible parent function. We also notice in the denominator we have $2x+3$. So $2x+3$ is another pssible function. So now we have to decide which one is $f(x)$ and which is $g(x)$. $g(x)$ is the function being *inserted* as the variable, so the inner function, and $f(x)$ is the function that is being inserted into, so the outer function. In our case, if we take $f(x) = \frac{1}{x}$ and $g(x) = 2x+3$, we should get our given function $y = \frac{1}{2x+3}$. Let's check that this is the case. Let $f(x) = \frac{1}{x}$ and $g(x) = 2x+3$. Now we'll verfiy that $f(g(x)) = \frac{1}{2x+3}$.

$$\begin{aligned} f(g(x)) &= f(2x+3), \text{ where } g(x) = 2x+3 \\ &= \frac{1}{2x+3} \end{aligned}$$

Therefore, we made the correct "guess"; $f(x) = \frac{1}{x}$ and $g(x) = 2x+3$.

Exercises

For each function determine f and g so that $y = f(g(x))$.

a) $y = (x+1)^2 - 2x(x+1) + 3$

b) $y = frac42x^2 + 3$

c) $y = -2\sqrt{x-1}$

d) $(x^2+3)^4$

e) $y = -(4x-5)^3$

f) $y = \sqrt{x+1}$

g) $y = \frac{-1}{x+1}$

h) $y = (x+1)^2$

i) $y = \sqrt{x^2+1}$

j) $y = (x^3+1)^4$