Completing the Square



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## What does "completing the square" mean?

First we need to understand what a "square" is. A square is a quadratic that is a *perfect square*. So it can be factored in the following way,

$$(x+a)^2\tag{1}$$

where a is any real number. If we expand or multiply out (1) we can see what a complete square looks like.

$$(x+a)^2 = (x+a)(x+a)$$
  
=  $x^2 + ax + ax + a^2$   
=  $x^2 + 2ax + a^2$  which is a perfect square.

To "complete the square" means we're given some part of

$$x^2 + 2ax + a^2 \tag{2}$$

and we have to "complete" it so we have

$$\frac{x^2 + 2ax + a^2}{x^2 + a^2} = (x+a)^2$$

somewhere in our quadratic. Why we want to complete the square will come later when we look at graphing of quadratics.

### Example

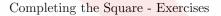
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Let's consider an example and go through the process of completing the square. Complete the square of the following quadratic,

$$x^2 + 6x$$

Solution What do we need to add to this to complete the square?

- 1. First, we know that 6 = 2a. This means that a = 3.
- 2. Second, we have to add  $a^2 = 3^2 = 9$ .





3. Third, we now have,

$$x^2 + 6x + 9 - 9$$

Notice that we added 9 and subtracted 9. The reason for this is we cannot just add a number to a function. If we did then we would be changing the function. So instead we add a 0. How we write 0 will make all the difference when completing the square. In this case since we need to add 9, we write 0 as,

$$a^2 - a^2$$
 or  $9 - 9$ 

4. Fourth, rewrite with a perfect square in the expression of the quaddratic. Once we add our strategicly written 0 we have,

$$x^{2} + 6x + 9 - 9 = (x^{2} + 6x + 9) - 9$$
  
= (x + 3)(x - 3) - 9  
= (x + 3)^{2} - 9

#### Example

Let's try another example. Complete the square of the following quadratic,

$$4x^2 + 7x$$

#### Solution

1. What part of (2) is missing or needs to be altered? We need to work with the form  $x^2 + bx$ . So let's factor out the 4 to get us into the required form.

$$4x^2 + 7x = 4\left(x^2 + \frac{7}{4}x\right)$$

2. Now, complete the square of  $x^2 + \frac{7}{4}x$  This means,

$$\frac{7}{4} = 2a$$
$$\frac{7}{8} = a$$

3. Add 0. We will be adding zero where our 0 is written as,

$$0 = a^2 - a^2 = \left(\frac{7}{8}\right)^2 - \left(\frac{7}{8}\right)^2 = \frac{49}{64} - \frac{49}{64}$$



This then gives us,

$$4\left(x^2 + \frac{7}{4}x + \frac{49}{64} - \frac{49}{64}\right)$$

4. Rewrite the original equation with as a completed square. Let's give this a try.

$$4\left(x^{2} + \frac{7}{4}x + \frac{49}{64}\right) - 4\left(\frac{49}{64}\right) = 4\left(x + \frac{7}{8}\right)\left(x + \frac{7}{8}\right) - \frac{49}{16}$$
$$= 4\left(x + \frac{7}{8}\right)^{2} = \left(\frac{7}{4}\right)^{2}$$

## Example

Let's try another example. Complete the square of the following function,

$$x^2 + 8x - 14$$

- 1. Is this a completed square? No
- 2. Identify the part that will be completed as a square. In this case it is  $x^2 + 8x$  will be completed as a square.
- 3.

 $\begin{array}{rcl}
8 &=& 2a \\
4 &=& a
\end{array}$ 

4. Add 0 where

$$0 = a^2 - a^2 = 16 - 16$$

This gives us,

$$(x^{2}+8x) - 14 = (x^{2}+9x + 16 - 16) - 14$$

5. Rewrite function as a completed square.

$$(x^{2} + 8x) - 14 = (x^{2} + 9x + 16 - 16) - 14$$
$$= (x^{2} + 8x + 16) - 16 - 14$$
$$= (x + 4)(x + 4) - 30$$
$$= (x + 4)^{2} - 30$$



Completing the Square - Exercises

# Exercises

Complete the square of the following quadratics.

a) 
$$x^2 - 12x$$
 e)  $x^2 + 5x - 10$ 

b)  $2x^2 - 7x + 10$ 

f)  $x^2 + 8x$ 

c) 
$$3x^2 - 4x$$
 g)  $-4x^2 + 8x - 12$ 

d)  $-6x^2 + 20x$