Circumcentre of a triangle

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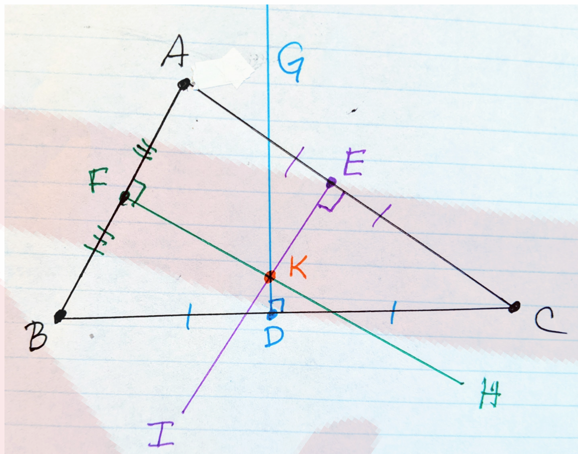
Circumcentre

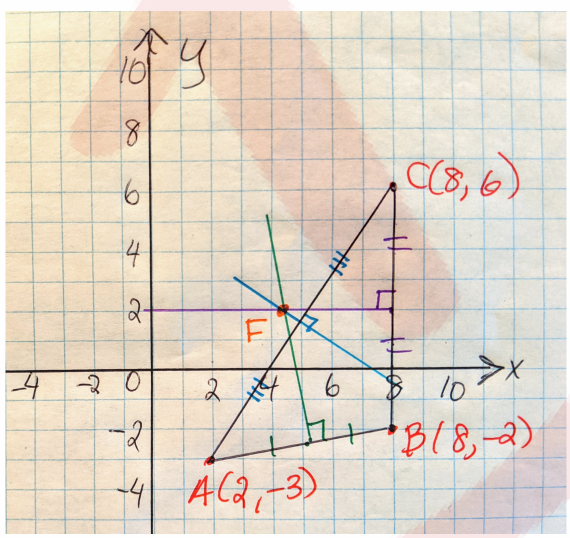
The circumcentre is the point where the three perpendicular bisectors of each side of a triangle intersect. The *perpendicular bisector* is the the line that is perpendicular to a side of a triangle and bisects that side.



In the diagram above, DG is the perpendicular bisector of BC; FH is the perpendicular bisector of the AB; EI is the perpendicular bisector of AC.

Example: Find the coordinates of the circumcentre of a triangle whose vertices are A(2,-3), B(8, -2) and C(8,6).

Solution: The first step is to draw a diagram of the triangle and it's coordinates.



The cirumcentre is the point of intersection of the three perpendicular bisectors of a triangles. If we can find the point of intersection of two of the perpendicular bisectors then we have our circumcentre. We need to find the equations of two of the perpendicular bisectors. Let's find the equation of the perpendicular bisector of AB. We need the slope of AB and the midpoint, D, of AB. Let's begin by finding the slope of AB.

$$\begin{aligned}\text{Slope of AB} = m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2+3}{8-2} \\ &= \frac{1}{6} \\ \therefore m_{\perp AB} &= -6\end{aligned}$$

Now for the midpoint, D, of AB.

$$\begin{aligned}\text{Midpoint of AB} = D &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{2+8}{2} m \frac{-3-2}{2}\right) \\ &= (5, -2.5)\end{aligned}$$

The equation of the perpendicular bisector to AB is given by,

$$\begin{aligned}y - y_2 &= m_{\perp AB}(x - x_)\text{, where } D(5, -2.5) = (x_0, y_0) \\ y + 2.5 &= -6(x-5) \\ y &= -6x + 27.5 \text{ is the equation of the perpendicular bisector of AB.}\end{aligned}$$

Let's find the equation of the perpendicular bisector to CB.

$$\begin{aligned}\text{Slope of CB} = m_{CB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2-6}{8-8} \\ &= \frac{-8}{0} \\ &= \infty \text{ or CB is a vertical line.} \\ \therefore m_{\perp CB} &= \frac{0}{-8} = 0 \text{ or, the perpendicular bisector to CB is a horizontal line.}\end{aligned}$$

The midpont of CB = E is given by,

$$\begin{aligned}E &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{8+8}{2}, \frac{6-2}{2}\right) \\ &= \left(\frac{16}{2}, \frac{4}{2}\right) \\ &= (8, 2)\end{aligned}$$

The equation of the perpendicular bisector of CB is,

$$\begin{aligned} y - y_0 &= m_{\perp CB}(x - x_0), \text{ where } E = (8,2) = (x_0, y_0) \\ y - 2 &= 0(x - 8) \\ y &= 2 \text{ is the equation of the perpendicular bisector of CB.} \end{aligned}$$

The point of intersection of the two perpendicular bisectors $y = -6x + 27.5$ and $y = 2$ is,

$$\begin{aligned} 2 = y &= -6x + 27.5 \\ 2 &= -6x + 27.5 \\ 6x &= 25.5 \\ x &= \frac{25.5}{6} \\ \therefore x &= 4.25 \text{ is the x-coordinate of the circumcentre.} \end{aligned}$$

Therefore, the circumcentre for ΔABC is $(4.25, 2)$.

Exercises

Find the circumcentre for the triangle with the following vertices:

a) (0, 4), (3, 6), (-8, -2)

b) (3, 2), (1, 4), (5, 4)

c) (1, 6), (5, 4), (5, -2)

d) (0, 3), (-4, -3), (2, -5)

e) (6, -1), (4, -5), (-2, -5)

f) (-2, 4), (2, 0), (-2, -4)