Centroid of a triangle

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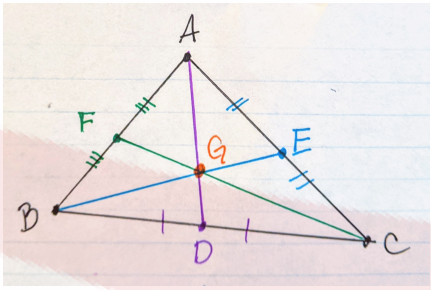
Centroid

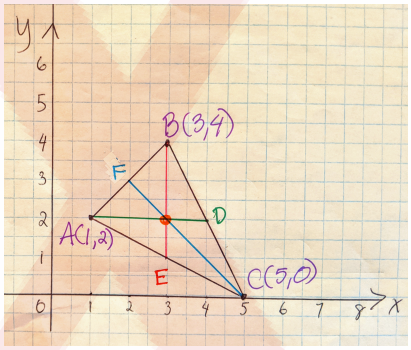
The *centroid* is the point where all three medians of a triangle intersect. The *median* is the line that joins a vertex of a traingle to the midpoint of the opposite side to the vertex.



In the diagram above, AB is the median of BC; BE is the median of AC; CF is the median of AB.

Example: Find the centroid of the triangle with vertices A(1,2), B(3,4) and C(5,0).

Solution: The first step is to draw diagram of the triangle and its vertices.



Remember, the centroid is the intersection of the medians of a triangle. So, we only need to find the intersection of two medians of a triangle to find the centroid. Let's find the median of side AB. We need to find the midpoint, F, of AB and then the slope of the line FC. Let's start by finding the midpoint F.

$$\begin{aligned} F &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{1+3}{2}, \frac{2+4}{2}\right) \\ &= \left(\frac{4}{2}, \frac{6}{2}\right) \\ \therefore F &= (2,3). \end{aligned}$$

We know the median of AB passed through F and C. The slope of the median is then given by,

$$\begin{aligned} m_{FC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-3}{5-2} \\ &= \frac{-3}{3} \\ &= -1 \end{aligned}$$

The equation of the line for the median of AB is given by,

$$\begin{aligned} y - y_0 &= m_{FC}(x - x_0) \text{ where } c = (5,0) = (x_0, y_0) \\ y - 0 &= -1(x-5) \\ \therefore y &= -x + 5 \text{ is the equation of the median of side AB.} \end{aligned}$$

Let's find the equation of the median of side AC. We need to find the midpoint, E, of AC.

$$\begin{aligned} E &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{1+5}{2}, \frac{2+0}{2}\right) \\ &= \left(\frac{6}{2}, \frac{2}{2}\right) \\ \therefore E &= (3,1). \end{aligned}$$

The slope of the median EB is given by,

$$\begin{aligned} m_{EB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-1}{3-3} \\ &= \frac{3}{0} \\ &= \infty \end{aligned}$$

Therefore, EB is a vertical line. The equation of the median of AC is then given by,

$$\begin{aligned} y - y_0 &= m_{EB}(x - x_0), \text{ where } E = (3,1) = (x_0, y_0) \\ x &= 3 \text{ since the equation of the median of AC is a vertical line.} \end{aligned}$$

The centroid is the point of intersection these two medians, $x = 3$ and $y = -x + 5$.

$$\begin{aligned} y &= -x + 5 \\ y &= -3 + 5 \\ y &= 2 \text{ is the y-coordinate of the centroid.} \end{aligned}$$

Therefore, the centroid of ΔABC is the point $(3, 2)$.

Exercises

Find the centroid of the trianbgles with the following vertices:

a) (-2, -3), (6, 7), (4, 1)

b) (1, -4), (-2, 2), (4, 5)

c) (0, -2), (4, 0), (2, 8)

d) (-6, 0), (0, 0), (0, 3)

e) (-5, 0), (2, 0), (0, 6)

f) (-5, 0), (0, -4), (1, 0)